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Biological Models for the New England Groundfish Fishery

by

Emma M. Henderson

National Marine Fisheries Service
Northeast Fisheries Center
Woods Hole, MA. 02543

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1. Introduction

The groundfish model has been designed with four requirements playing major roles. First, we looked for means of satisfying an extensive list of criteria presented by the New England Fishery Management Council staff (Henderson and Marchesseault, 1979). Second, the model was planned to use existing data bases and analyses in order to be operational sooner and also to ensure continuity with historical values. Third, the assumptions about fishery practices and about the biology of each species were chosen to suit the New England groundfish fishery as closely as possible. Fourth, the biological model has been planned to interface with future economic models. The current model is a first cut at satisfying each of these points and can be modified to include new points, new parameter values, or to provide alternate outputs as these changes become desirable and practical.

The model predicts stock variables for a number of years with specified annual removals and initial values. The variables calculated by the model are numbers and biomass of stock, spawners and recruits at annual intervals. The model also calculates the probability distribution of each variable at the end of the period. Growth is calculated from standard length-age and weight-length curves. Mortality and yield are calculated from the usual population dynamics equations. New methods are introduced for projecting future year-class strength and partial recruitment patterns, each matched to historical data. Year-class strength is calculated from a spawner-recruit equation, with the option of including a random factor. The random fluctuations in recruitment induce random variation in each variable, leading to the required probability distributions. Methods for estimating all required parameters are included.

The uses of the model are many and varied. A proposed set of annual quotas for the groundfish stocks can be examined for their effect on future yields, for their effect on subsets of the stock, such as spawners or new recruits, or for changes in mean weight of landed fish. The probability of achieving proposed

yields, stock levels or stock composition can also be assessed. Changes in standard practice can be introduced and simulated, for example, changes in gear or the effects of seasonal quotas.

The model format and assumptions are discussed in more detail in section 2.1, computer programs in section 2.2. Derivation and estimation of partial recruitment coefficients are treated in section 2.3. Derivation and estimation of spawner-recruit coefficients and random pre-recruit mortality are discussed in section 2.4. Section 3, Discussion, treats validation of the model, planned uses of the model and possible extensions.

Section 4 describes a bio-economic application in which the set of proposed annual quotas is obtained from optimizing a set of economic considerations (Vidaeus and Marchesseault, 1978). Seven groundfish stocks are assumed to be linked through the economic interactions of ex-vessel prices. The biological model provides linear constraints for quadratic programming software and also assesses the probability that management objectives will be satisfied by the resulting quotas.

2.1 Model format and logic.

The model is a single species simulation model with annual time steps. Briefly, the recruited stock is divided into classes according to weight rather than age. It is assumed that fish are recruited upon reaching a threshold weight. Each weight class represents one year's growth. Thus, new recruits enter the first class and progress one class per year. In the initial year the number of fish in each class is determined from fisheries data. In subsequent years the model calculates the changes due to natural and fishing mortality losses, increase in weight and added recruitment. Each cohort contributes new recruits over a period of several years. Initial cohort strength is calculated from a spawner-recruit equation where spawners are a specific subset of the total stock. The equations representing the above statements are summarized below, with more detailed comments following.

$$N_t = \sum_{k=1}^{K} N_{kt} \tag{1}$$

$$B_t = \sum_{k=1}^{K} N_{kt} w_k \tag{2}$$

$$B_t^* = \sum_{k=1}^{K} N_{kt} \bar{w}_k \tag{3}$$

$$\mu_t = \frac{Y_t}{B_t^*} \tag{4}$$

$$s_t \doteq \alpha + \beta \mu_t \tag{5}$$

$$Z_t = \ln s_t \tag{6}$$

$$F_t = Z_t - M \tag{7}$$

$$NS_t = \sum_{k=km}^{K} N_{kt} s_t^{\tau} \tag{8}$$

$$BS_t = \sum_{k=km}^{K} N_{kt} w_{\tau k} s_t^{\tau} \tag{9}$$

$$N_{kt} = N_{k-1,t-1} s_{t-1} \qquad k=2,\ldots,K-1 \tag{10}$$

$$N_{Kt} = (N_{K-1,t-1} + N_{K,t-1}) s_{t-1} \tag{11}$$

$$A_{rt} = (C_1 NS_{t-r} + C_2 BS_{t-r}) e^{-\varepsilon_t} \tag{12}$$

$$N_{1t} = \sum_{i=o}^{R-r} \pi_i A_{r,t-i} \tag{13}$$

The K weight classes are defined by K+1 weights, $w_1, w_2, \ldots, w_K, w_\infty$. Fish in the kth class grow from w_k to w_{k+1} during the year. The last class, (w_K, w_∞), contains all fish with weight equal to or exceeding w_K. An estimate of w_1 is obtained by observing the minimum length consistently appearing in commercial length-frequency data. Annual increments in length are calculated from the von Bertalanffy equation in the form

$$L_{t+1} = L_\infty(1-e^{-\kappa}) + e^{-\kappa}L_t$$

Corresponding weights are calculated from the weight-length equation

$$w=aL^b$$

The parameter K is chosen for each stock to give a convenient set of weights.

N_{kt} is the number of fish in the kth class at the beginning of year t.

N_t is the total number of recruited fish at the beginning of year t. For the first year, t=1, commercial length-frequency data for the preceding period give estimates of the ratios N_{k1}/N_1. $\hat{N}_1$ = (total catch) ÷ (exploitation rate) with the exploitation rate corresponding to the $\hat{F}$ in the assessment for that year.

B_t is the biomass of recruited stock at the beginning of year t.

Y_t is the annual yield, or quota, during year t. The values of Y_t are input for each year in the planning period.

μ_t is the corresponding annual exploitation rate on recruited fish in year t. This exploitation rate is independent of size (and age).

B_t^* is the effective biomass of recruited stock during year t. B_t^* has been defined to enable use of equation (4). $\bar{w}_k$ is the average weight of a fish in class k during the year. The current estimate of $\bar{w}_k$ is the weight at midyear.

s_t is the annual survival rate of recruited fish in year t. Equation (5) was developed to avoid iterative solutions for s in terms of μ. Pairs of values of $s=e^{-(F+M)}$ and $\mu = \frac{F}{F+M}(1-e^{-(F+M)})$, $0 \leq F \leq 1$, were calculated to

serve as "observed values" for estimating α and β by least squares ($\hat{\alpha}$= .81745, $\hat{\beta}$ = .88953, r^2= .99997 for M=.2 and $.3 \leq s \leq e^{-.2}$).

M is the instantaneous natural mortality rate on recruited fish. M is an input parameter and is assumed constant.

Z_t is the instantaneous total mortality rate on recruited fish in year t.

F_t is the instantaneous fishing mortality rate in year t. Z_t and F_t are not needed in the model, but can be calculated if of interest. Equations (4) - (7) can also be modified to allow using F_t as input.

Spawners are determined by reducing numbers at the beginning of the year by losses up to spawning time, using weights at spawning time and summing only over classes containing mature fish.

NS_t is the number of spawners in year t.

BS_t is the biomass of spawners in year t.

τ is the peak spawning date expressed as a fraction of a year. It is assumed that the single date τ can be substituted for the actual spawning period which may occupy several months.

km denotes the first class containing mature fish. τ and km are estimated from general information about each stock.

$w_{\tau k}$ is the weight of a fish in class k at spawning time. $w_{\tau k}$ is currently estimated from the growth curves.

After the above quantities have been calculated, t is advanced one year and new values of N_{kt} are calculated using equations (10) - (13). Steps (1) - (13) are then repeated for each year in the planning period.

r is the age at which fish are first recruited to the fishery.

R is the age at which fish are first fully recruited to the fishery.

A_{rt} is the abundance of r-group fish at the beginning of year t. It is only necessary to calculate abundance at one age, r.

N_{1t} is the number of new recruits entering the fishery during year t.

N_{1t} is the sum of new recruits of ages r to R in year t, expressed as fractions π_i of the contributing year classes $A_{r,\ t-i}$. Estimation of π_i is discussed in section 2.3.

Equation (12) and estimation of the coefficients are discussed in section 2.4. In the deterministic mode $\varepsilon_t \equiv 0$. In the stochastic mode ε_t is generated from random normal variable software with mean $\bar{\varepsilon}$ and standard deviation s_ε. If recruitment is assumed to be independent of spawning stock, equation (12) is replaced by $A_{rt} = C_o e^{-\varepsilon t}$.

2.2 Computer Programs.

It appeared simpler to write separate programs for the deterministic and stochastic options because of differences in output formats and internal storage requirements. The programs are written in FORTRAN-IV for the Xerox Sigma-7 computer.

The model is a single-species model. Input is parameterized so that numerical values for each species can be read in as data. Units are kilograms for individual fish, thousands of fish or kilotons for total stock, and years. Input includes a stock ID code and values for age at first recruitment, r, age of full recruitment, R, spawning date, τ, number of weight classes, K, first spawning index, km, average weights, $\{\bar{w}_k\}$ spawning weights, $\{w_{\tau k}\}$, initial frequencies $\{N_{k1}\}$, year-class abundances from $A_{r,r-R+1}$ to $A_{r,r}$, natural mortality rate, M, spawner-recruit parameters, C_0, C_1, C_2, and partial recruitment coefficients, $\{\pi_i\}$. The stochastic program requires in addition the parameters $\bar{\varepsilon}$ and s_ε. General input includes the number of years to be run, T, the first calendar year and the set of annual quotas, $\{Y_t\}$. Generalized dimensions have been used for storages.

The deterministic program, DET, outputs annual values of N_t, B_t^*, NS_t, BS_t, A_{rt}, N_{1t}, Y_t, Z_t, F_t and μ_t. The calculations have been described in section 2.1.

The stochastic program, STO, provides the probability distributions and related parameters for stock values at the end of the planning period. Frequency classes are set up for three output variables, numbers of total stock, N_t, spawning stock, NS_t and incoming recruits, N_{1t}. The range of the frequency classes covers the expected range of the variable for the species being modeled. The class width is chosen to give as much accuracy in output as is consistent with obtaining smooth probability curves. Smoothness also depends on the number of trials. Random trials are simulated by using standard software to generate random normal variables, ε, with mean $\bar{\varepsilon}$ and standard deviation s_ε (see section 2.4). In each year A_{rt} is multiplied by a factor $e^{-\varepsilon_t}$. One trial consists of a complete planning period of T years and NT trials are run. Currently NT=300. At the end of T years, probabilities of occurrence are cumulated by adding 1/NT to the appropriate frequency class for each variable. In addition, the program keeps track of the maximum and minimum value attained by each variable. After the NT trials are complete, the probabilities are used to calculate the expected value and variance for each variable. The formulas used are:

$$E(X) = \sum_{i=1}^{NT} x_i p(x_i)$$

$$V(X) = \sum_{i=1}^{NT} (x_i - E(X))^2 \, p(x_i)$$

where x_i is the mid-point of the ith class interval. Output includes the probability distribution, expected value and variance, maximum and minimum for each variable. Both programs print out the input data and control variables used in that run.

The programs are still in a developmental stage. Both programs have been debugged using an arbitrary set of parameters and initial data, and with F_t as input rather than Yt. It remains to improve the input data file system, add computer graphics and provide complete program documentation. Further experimentation with choosing class intervals, number of trials,

and random number parameters to achieve reasonable stochastic output is also desirable so that guidelines can be established for the operator.

In addition to the computer programs, the author has written programs for an HP-97 desk calculator for estimating the various parameters described here.

2.3 Partial Recruitment

We wish to estimate the number of fish recruiting to the fishery in a given year. If fish are first recruited at age r and fully recruited at age R, one cohort will contribute a certain number of recruits at age r the first year, a certain number at age r+1 the following year, etc. It is convenient to calculate the number of recruits for each year in terms of the cohort abundance the first year. To obtain the recruits in a single year we then sum the age r recruits from the appropriate year-class, age r+1 recruits from the preceding year-class, etc.

For the calculations we require catch at age data for several cohorts and the exploitation rates. To satisfy the model assumption that exploitation rate is independent of size (i.e. age), we use μ_t, the common exploitation rate on fully recruited fish in year t. The methods used here are a modification of those presented by Allen (1966). Definitions are:

t = year in which a cohort reaches age r

A_{rt} = abundance of r-group fish at the beginning of year t

$R_{r+i,t+i}$ = number of (r+i)-group fish recruiting in year t+i

$$\pi_{it} = \frac{R_{r+i,t+i}}{A_{rt}} \qquad i=0,\ldots, R-r \qquad (14)$$

= the fraction of the cohort recruiting at age r+i in year t+i

$N_{r+i,t+i}$ = total number of recruited (r+i)-group fish in year t+i

It follows from the definitions that

$$N_{rt} = R_{rt} \qquad (15)$$

$$N_{r+1,t+1} = N_{rt}s_t + R_{r+1,t+1}$$

$$\vdots$$

$$N_{\quad} = N_{\quad} s_{\quad} + R_{\quad}$$

Equations (15) allow calculating the R's from the data. Substituting $Nit = \frac{C_{it}}{\mu_t}$ and rearranging gives

$$R_{rt} = \frac{C_{rt}}{\mu_t}$$

$$R_{r+i,t+i} = \frac{C_{r+i,t+i}}{\mu_{t+i}} - \frac{C_{r+i-1,t+i-1}}{\mu_{t+i-1}} s_{t+i-1} \qquad i=1,\ldots,R-r$$

We can now calculate the corresponding π_{it}'s from equation (14) by dividing the R's by an estimate of A_{rt}.

We now have a set of π_{it} for each cohort, indexed by t. Since it is desirable to have coefficients independent of t for projection into the future, the estimates of π_{it} are averaged over t for use in the model.

$$\hat{\pi}_i = \bar{\pi}_{it} \qquad t \text{ in data set}$$

We have also established equation (13) in section 2.1

$$N_{1t} = \sum_{i=o}^{R-r} \pi_i A_{r,t-i} \qquad (13)$$

The model uses the average historical pattern. Large deviations from the average should be studied by choosing special coefficients to represent the situation of interest. We also note that discarding of partially recruited fish as well as other mortalities acting on partially recruited fish are included in the π's.

2.4 The spawner-recruit equation.

We developed a generalized spawner-recruit equation that would express the dependence of recruits on spawners and on chance factors to the extent permitted by the available data. That data comprises estimates of numbers and biomass of spawners and recruits for a period of years. If the dependence on spawners is not estimable, the equation reduces to one that is independent of spawners. If the dependence on chance factors is omitted, the model becomes deterministic rather than stochastic. The generalized equation is

$$A_{rt} = (C_0 + C_1 NS_{t-r} + C_2 BS_{t-r}) e^{-\varepsilon t}$$

In usage the options mentioned above are chosen to suit the species, for example, setting $C_1=C_2=0$ specifies that recruits are independent of spawning stock. A justification for including both numbers and biomass is given below, as well as estimation methods for the C's and ε's.

Abundance of age r fish is the product of two factors.

A_{rt} = (total eggs spawned in year t-r) x (survival from t-r to t)

The first factor is calculated using the premise that fecundity is linear in weight (Oosthuizen and Daan, 1974) with slope ϕ and intercept w_α.

eggs per i^{th}♀ = $\Phi\ (w_i - w_\alpha)$

$$\text{total eggs} = \sum_{\text{spawning ♀'s}} \text{eggs per ♀}$$

$$= \phi \times \text{biomass of spawning ♀'s} - \phi w_\alpha \times \text{number of spawning ♀'s}$$

The second factor is obtained by assuming that mortality from age 0 to age r is a normal random variable.

$$M_{rt} = \bar{M}_r + \varepsilon_t \qquad \varepsilon_t \sim N(0,\sigma_\varepsilon^2)$$

Then survival to year t is a log-normal random variable, $e^{-M_{rt}}$. Multiplying the two factors gives A_{rt} in the form of equation (12) in section 2.1.

$$A_{rt} = (C_1 NS_{t-r} + C_2 BS_{t-r})\ e^{-\varepsilon_t} \qquad (12)$$

Since the coefficients C_1 and C_2 have absorbed $e^{-\bar{M}_r}$, ϕ, w_α and the ♂:♀ ratio, these quantities need not be estimated individually. ($C_1 = -\phi w_\alpha \frac{N_♀}{N_{tot}} e^{-\bar{M}}$ and $C_2 = \phi \frac{B_♀}{B_{tot}} e^{-\bar{M}}$).

It is assumed that these parameters are all independent of t. $\hat{C}_1$ and $\hat{C}_2$ are obtained by minimizing the sum of squares

$$\Sigma\ \varepsilon_t^2 = \Sigma\ (\ln\{(C_1\ NS_{t-r} + C_2\ BS_{t-r})/Art\})^2$$

using the available data set for spawners and recruits. If NS_{t-r} and BS_{t-r} were not measured at spawning time they should be reduced by the factor s_{t-r}^{τ}. Then

$$\varepsilon_t = \ln\{(\hat{C}_1\ NS_{t-r} + \hat{C}_2 BS_{t-r})/A_{rt}\}$$

The set of $\hat{\varepsilon}_t$ yields $\bar{\varepsilon}$ and s_ε, the parameters that describe the probability distribution of random pre-recruit natural mortality.

In the alternate form

$$A_{rt} = C_0 e^{-\varepsilon t}$$

non-linear regression gives

$$\hat{C}_0 = \text{geometric mean of } \{A_{rt}\}$$

$$\hat{\varepsilon}_t = \ln(\hat{C}_0/A_{rt})$$

Thus, the parameters describing deterministic recruitment and stochastic fluctuations are based on historical data and can be used to project the "same" patterns into the future. Because of the properties of the log-normal, the deterministic values obtained by setting $\varepsilon_t \equiv 0$ fall between the mean and the mode of the distribution.

3. Discussion

In this section we shall discuss the current status of the groundfish model, methods of validation, and uses of the model for problems in the groundfish fishery.

This report constitutes a description of the current status of the model. To summarize, the theory has been based on an initial set of premises sufficient to satisfy basic needs, equations and numerical methods have been developed, and computer programs written and debugged.

Numerical validation will consist of verifying predicted stock behavior by comparing new data values with model output. Particular comparisons will have to take into account uncertainties in the data being used. Data estimated more directly, such as mean weight in the commercial catch, or commercial and survey length frequencies, would provide better checks than, say, estimates of year-class strength, which are not firm until several years after recruitment. Validity of the stochastic model can be tested by seeing if new values fall within confidence bands about the predicted values, and if a longer time series yields the same distributions that were fitted to historical

data.

Output of the model can also be compared with other models, such as yield-per-recruit. A study of any discrepancies might yield useful insight. Some discussion is given in Beverton and Holt, 1957. The groundfish model is more flexible as it does not require a constant F or any equilibrium assumptions, treats recruitment as a predictable variable, and allows recruitment over several years rather than at a single point in time.

The model is not intended for long range prediction, or for situations radically different from those reflected in the data base. However, once basic validity is established, new situations of interest can be simulated and studied.

Three points are not treated by the current model since the data base does not contain corresponding assumptions or analyses. However, the model could be modified to simulate these conditions for preliminary studies and future inclusion. First, seasonal factors are not modeled. These include factors which do not occur uniformly throughout the year, such as seasonal quotas or seasonal fisheries, spawning, and recruitment. January 1 has been used for "the beginning of the year". The other two omissions are dimorphism and size selective gear.

Also, for the initial model other points have been simplified or have been treated with trial methods. All omissions and invalid assumptions contribute to the final variance. As validation progresses each of these points can be tested for importance and developed further if warranted. There is no apparent difficulty in modifying the model to handle the cases cited.

In the present format the model can be used to study the effects of proposed annual quotas over a multi-year period. The detail with which many factors are treated should provide a great deal of insight into cause and effect of changes in the stocks. Thus, general guides to management

could be established, as well as the effects of deviating from those guides. Planned changes might include new minimum size regulations or closed seasons, while changes in discard practices would be unplanned. Readjustment of quotas to correct for these deviations could also be studied. Achievable goals could be identified as well as danger points to be avoided.

Similar modeling methods could be used to investigate many related problems such as allocation of quotas to multiple fisheries, effects of changes in proportions of gear types, bycatch analyses, or estimating the potential of undeveloped fisheries.

4. Bio-economic applications.

Although the bio-economic model described in this section (Vidaeus and Marchesseault, 1978) represents a particular example, the proposed methods illustrate the generality of possible applications using the simulation model to provide the proper biological background, to tune an approximate linearized model to the more accurate model, to provide stochastic analyses, and to demonstrate stock behavior over a range of proposed strategies, not just at an optimum point.

The particular application under study can be stated in general terms as follows. Since ex-vessel prices for each species are determined in part by the mix of species landed, economic goals of management can be stated as a quadratic function of the annual quotas for a multi-year planning period. Maximizing the present value of this function leads to a set of proposed "optimum" quotas, but the economic optimum must also satisfy two other criteria. The proper biological conditions must be maintained for each stock and there should be a reasonable probability for achieving the desired conditions under the proposed plan. Standard mathematical techniques for solving this type of problem are referred to as quadratic programming optimization, or QPO. The available QPO software has added some operational conditions. First, any biological constraint, for example, minimum spawning

stock to be permitted, should be stated as a linear function of the annual quotas. Second, the constraints should be deterministic, not stochastic.

These interfacing problems can be solved by the following procedures. Although the equations presented in section 2.1 would lead to non-linear constraints, the equations for biomass of total stock and for biomass of new recruits can be modified to a linear format by introducing the extra assumptions and approximations discussed later in this section. Constraints would be stated as $B_t \geq$ BMIN or $BR_t \geq$ BRMIN. Preliminary numerical values for BMIN and BRMIN to ensure particular stock conditions would be determined by using the simulation model in a deterministic mode with some trial quotas. The resulting linear constraints and the quadratic economic objective function become the input for the QPO software. The proposed set of optimum quotas output by the QPO model become input for the non-linear simulation model in the stochastic mode for assessing the probability that all desired management goals are met. It is expected that some iteration of the procedure may be required to obtain a set of constraints producing the final set of quotas.

The approach described here allows the derivation of two linear equations to be used in formulating constraints.

$$B_t = a_{ot} + \sum_{i=1}^{t-1} a_{it} Y_i \qquad t=1,\ldots,T \qquad (16)$$

$$BR_t = b_{ot} + \sum_{i=1}^{t-2} b_{it} Y_i \qquad t=1,\ldots,T \qquad (17)$$

where B_t and BR_t are the biomass of total stock and of new recruits at the beginning of year t, and Y_t is the quota for year t. The planning period covers T years. The author was unable to derive usable linear expressions for numbers of fish.

In the notation of section 2.1 $BR_t = N_{1t}w_1$. Equation (13) allows calculating N_{1t} from the year-class abundances formerly given by equation

(12), but now equation (12) must be replaced by

$$A_{rt} = C_1' \, BR_{t-r} + C_2' \, B_{t-r} \tag{12'}$$

since the formulation is restricted to B_t and BR_t. The final result is an expression for BR_t as the sum of terms in BR_t and B_t for prior years.

$$BR_t = w_1 \sum_{i=o}^{R-r} \pi_i (C_1' \, BR_{t-r-i} + C_2' \, B_{t-r-i}) \tag{18}$$

Similarly, B_t^* in equation (4) must be replaced by a suitable expression. Substituting equations (10), (11), (13) and (5) in equation (2) gives

$$\begin{aligned} B_t &= \sum_{k=1}^{K} N_{kt} w_k \\ &= N_{1t} w_1 + \sum_{k=1}^{K} N_{k,t-1} w_{k+1} s_{t-1} \\ &= BR_t + (\sum_{k=1}^{K} N_{k,t-1} w_{k+1})(\alpha + \beta \mu_{t-1}) \end{aligned}$$

We now use the two empirically established approximations

$$\sum_{k=1}^{K} N_{k,t-1} \, w_{k+1} \doteqdot \Gamma^2 \, B_{t-1}$$

$$\sum_{k=1}^{K} N_{k,t-1} \, w_{k+1} \doteqdot \Gamma B_{t-1}^*$$

along with equation (4) to obtain

$$B_t = BR_t + \alpha \Gamma^2 B_{t-1} + \beta \Gamma B_{t-1}^* \mu_{t-1}$$

$$B_t = BR_t + \alpha \Gamma^2 B_{t-1} + \beta \Gamma Y_{t-1} \tag{19}$$

An estimate of Γ is obtained by averaging the T values of

$$\hat{\Gamma}_t = \frac{B_t^*}{B_t} \qquad t=1,\ldots,T$$

obtained from the original model. A heuristic argument for introducing Γ is that $\bar{w}_k$ is approximately the geometric mean of w_k and w_{k+1}.

The two recursion relations (18) and (19) yield equations (16) and (17) by repeated substitution for B and BR on the right hand side. The coefficients, a_{it} and b_{it}, are complex expressions involving the model parameters and initial data, so are specific to calendar year as well as to stock. The calculation of the a's and b's can be simplified by using recursion relations

based on (18) and (19) (details omitted).

The linearized model was tested using the debugging data and appeared satisfactory by the criterion that linear and non-linear models should give similar output.

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