A simulation study to evaluate estimation of biological reference points from VPA and ASAP

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A Working Paper in Support of Term of Reference 1

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GARM 2008 Biological Reference Point Meeting
Woods Hole, MA
28 April – 2 May 2008
Executive Summary

A simulation study was performed to evaluate two NOAA Fisheries Toolbox assessment models (VPA and ASAP) with respect to their ability to estimate biological reference points (BRPs) and the parameters of a stock recruit function. Data sets with different lengths of time, three different levels of recruitment variability (0%, 20%, and 80% CV), and two levels of steepness (h=0.60, 0.88) were simulated with PopSim, a simulation program in the Toolbox. Each simulated dataset was fit in the VPA and in ASAP. The estimated time series of spawning biomass and recruits from each model were passed to SRFIT, another Toolbox program, to estimate the stock recruit function and the corresponding BRPs. These externally estimated reference point values were compared to the true values to determine bias and precision. In addition, the internally estimated BRPs from ASAP were compared to the true values.

Between externally estimated BRPs from VPA and ASAP runs on the same data sets, the bias in estimates of the stock-recruit parameters was similar, but slightly less for ASAP, which carried through to less bias in the BRPs. Comparing internally versus externally estimated stock-recruit parameters for ASAP, the external estimates of $R_0$ were generally less precise, but slightly less biased for CV=0% and CV=20%. However, the bias in external estimates was quite severe when CV=80%. This may relate to misspecification in the default level of recruitment variability assumed in ASAP and SRFit; it would require further detailed tuning to evaluate the impact of that model setting. When the ASAP model was applied to data from three different time periods with different amounts of data in each period, we found that the model performance was improved by extending the series as far back as there were indices (1963), but extending back to 1935 when only total catch was available produced no gain and oftentimes exacerbated the bias. For the VPA model runs using catch at age data that started in 1977 or in 1995, the shorter time series (only 12 years of data) did a very poor job of estimating unexploited levels of recruitment. This could be due to the length of the time series, or to the limited amount of contrast in stock size (the depletion level in SSB was pretty flat over that time period, ranging from 6% to 16%). Although there is not time to fully evaluate these hypotheses, based on the cases explored in this simulation, we conclude that short time series from an overfished stock are likely to produce informative time series of SSB and recruitment from which to estimate BRPs.

In all comparisons, the pattern of bias and precision in steepness carried through to the bias and precision of $F_{MSY}$ while unexploited recruitment ($R_0$) largely determined the bias and precision in MSY and $SSB_{MSY}$. 
Introduction

The general focus of this simulation was to evaluate bias and precision in biological reference point (BRP) estimates. Several specific points that we considered in the simulation were the comparison of internal versus external reference point estimation, whether estimated trajectories of recruitment and spawning stock biomass (SSB) from VPA and ASAP would produce similar estimates of BRPs and stock recruitment parameters, and whether the ability to incorporate historic removals improves the ability to estimate BRPs.

In comparing the externally estimated BRPs between the Toolbox implementation of VPA and ASAP, we were interested in evaluating whether structural differences between the two models would generate different estimates of SSB and recruitment, and whether the estimates from one model would lead to better estimates of reference points.

We addressed the suggestion that extending the time series of observation further back in time could provide additional contrast that would enhance the ability to estimate a stock recruit relationship. The ability of forward projecting statistical catch at age models to incorporate years where only total catch, or total catch and abundance indices, are available is touted as being an advantage over the VPA approach, which requires catch at age in all years. To test this hypothesis, we considered three time periods with varying degrees of data complexity. Comparisons of ASAP BRP estimates between the three data stanzas allowed us to draw conclusions about the usefulness of extended timeseries when the data composition is as simulated here.

Finally, we compare BRP estimates that are derived internally in ASAP, as opposed to the external estimates from application of SRFit.

The population simulator (called PopSim in the NFT) was specifically designed to work seamlessly with the NFT stock assessment programs and to allow comparison of identical datasets among models. Random noise is added to known conditions to create multiple realizations from a given scenario. Each random realization contains the same information for each of the models and therefore comparisons can be made directly.

As in all simulation work, there are many limitations to the conclusions that can be drawn from this study. Due to the automatic nature of the simulations and stock assessment, there is limited ability to “tune” each stock assessment model to a particular dataset. Each model is therefore set to “typical” conditions, and the result from each data set is “final” in the sense that the analyst does not have the ability to examine diagnostics and re-configure or re-run the model. Additionally, the PopSim model is limited in the types of uncertainty that can occur in the datasets. Specifically, there are no spatial components, sex-specific differences, density-dependent effects, nor calculated management interventions. However, even with these limitations, PopSim provided an excellent framework for this study and some general conclusions were reached. It should be noted, however, that PopSim is mechanistically more similar to ASAP, which could enhance ASAPs performance.
PopSim

PopSim is both an age and length based simulator. The user defines:

- dimensions of the scenario in terms of number of years, ages, and lengths
- initial stock abundance at age
- recruitment values or a stock recruitment relationship
- annual length based fishery selectivities and fishing intensities (separable F)
- biological characteristics
  - natural mortality
  - growth, length-weight, and maturity equations
  - variability in growth
- market categories for sampling length distributions
- annual sampling intensities by market
- ageing precision matrix
- survey characteristics (common to all stock assessment models)
- index characteristics (specific to stock assessment models).

Standard fishery science equations are used throughout the model to generate population abundance at age, catch, and survey values for the true conditions. These true values are used as the basis for comparison with stock assessment model estimates from 100 random realizations of the true data. There is the possibility of a small amount of bias being introduced between the true data and any assessment model—even if the dynamic equations were identical—because only 100 data sets were examined. Nevertheless, this was a feasible amount of data to examine for this meeting.

Because our goal was to evaluate model performance in the estimation of reference points, and information content from using different time period lengths and types of data, we avoided adding any additional uncertainty or model misspecification. For simplicity, we assumed a single selectivity vector (flat-topped) for all years, a single maturity ogive for all years, and constant natural mortality for all ages and all years. There are two input standard deviations which determine the uncertainty in growth. The first gives the spread of lengths about the initial population numbers at age as well as the spread of all future recruitments (age 1 in PopSim). The second standard deviation is used to create a growth transition matrix for each age. The expected growth from one age to the next is based on calculated lengths at age from the von Bertalanffy equation and then a normal distribution about this expectation is created from the second growth standard deviation, with the limitation that fish are not allowed to decrease in size. The normal distributions from all possible starting lengths at age are summed to create the growth transfer matrix for that age. We set these two standard deviations as low as possible (to PopSim’s lower limit), in order to avoid introducing artifactual patterns that would be due to something other than the method used to estimate BRPs.

Variability in the realizations provided to the stock assessment models is incorporated in PopSim in a number of ways. The annual total catch in weight values and recruitments are lognormally distributed. Noise is added to each age of each survey and applied to all lengths within the age according to a lognormal distribution. The indices used by a
specific stock assessment model are formed by summing the surveys with noise over length, age, or both. For example, if a survey has ten ages with 20% CV at each age, a total biomass index would be formed by summing all ages and lengths for a year times the weight at length. Lengths are sampled from the catch according to market category. Sampling within market categories was high so that total catch would be well characterized. The sampled fish are expanded to the total catch based on the weight of the sample and the total weight of the catch.

Cases Evaluated

One hypothetical stock (with Georges Bank yellowtail flounder-like life history), was the basis for all simulations. The biological parameters used to simulate the datasets are listed in Table 1, and Figure 1 provides a graphical overview of the different lengths of time series and data types used in each model.

For each case in this study, 100 “data sets” were generated and run through each assessment model (VPA v2.7.7 and ASAP v2). A Beverton-Holt stock recruit function was assumed, and three levels of recruitment variability were considered: CV=0%, CV=20%, and CV=80%. Another factor in this simulation was the steepness \( h \) parameter in the stock recruit function. We performed simulations at two steepness levels: \( h=0.88 \) and \( h=0.6 \). These two levels were chosen because they probably bound a likely range for many of the stocks that compose the groundfish complex. As is indicated in Figure 1, there are three time periods of data that were defined to mimic some of the data environments for stocks assessed as part of the groundfish complex. Catch at age data is passed to the assessment models for the years 1977-2006. Total catch and survey indices are available back to 1963, and total catch only is available back to 1935. We therefore considered 18 different assessment scenarios for ASAP (3 recruitment CV levels, 3 time periods, 2 steepness levels) and 12 scenarios for VPA (3 recruitment CV levels, 2 time periods, 2 steepness levels) with 100 “data sets” for each scenario for a total of 3000 total “assessments”.

PopSim was set-up to begin in 1935 at unexploited conditions. The simulated stock was fished at a rate that achieved similar depletion levels for each of the two steepness cases (Figure 2). By 1963, the stock was depleted to about 17% of unexploited conditions, and in the final year, the stock was depleted to less than 8% of unexploited conditions. By forcing such severe depletion by 1963, we are better able to test the hypothesis that incorporating historic landings will provide more contrast and better estimates of BRPs and stock recruit parameters.

VPA set-up

Data passed to the VPA model included catch at age, weights at age of the catch and the stock, maturity, natural mortality, and two indices of abundance for each age. An assessment using the VPA framework was performed for years 1977-2006, and also for
years 1995-2006. The reason for exploring the short time series is to evaluate the suggestion that, in the presence of a retrospective pattern, one might truncate the data series to a subset of years that are “retro-free” and base management advice on this time series. Although we did not introduce any misspecification that would cause a retrospective pattern (and indeed, none was observed), we performed this short time series assessment to see if there would be enough information and contrast in the data from which to estimate reference points. Estimated SSB and recruitment trajectories were imported to SRFit, and stock recruitment parameters and biological reference points were estimated.

**ASAP set-up**

The same data that was passed to the VPA was passed to ASAP. In addition, there were a number of CVs and weight options that could be specified for data components in ASAP. Every attempt was made to configure a standard assessment set-up, with objective rationales for specifying weights. For instance, initial test runs revealed that without a small penalty on unexploited SSB for the 1977-2006 runs, the solution tended towards 10^{25}. Likewise, for the 1935-2006 runs, a small penalty was imposed to keep the model close to match the ‘unexploited’ condition of the stock in 1935.

ASAP was run for all three time periods (1935-2006; 1963-2006; 1977-2006) to test the information gain by incorporating historical data for years where no catch at age is available. Additionally, because ASAP can estimate a stock recruit function and reference points internally, a comparison was made between internally estimated reference points and estimates derived from externally fitting a stock recruit function to ASAPs estimated trajectories of SSB and recruitment. The external fit was performed with SRFit.

**SRFit set-up**

Estimation of the stock-recruit relationship and the biological reference points in SRFit was automated so that assessment model output streams of SSB and recruitment, and the final year of age specific maturity, catch weight, stock weight, and selectivity, were pasted into a template that contained all remaining specifications. The remaining specifications for the SRFit template were the form of the stock recruit function (Beverton-Holt), initial guesses for the stock recruit parameters and the error variance associated with recruitment variability (0.7). No priors or other penalties were specified.

**Estimation of true reference points**

For each case considered, and for each of the 100 iterations within those cases, the values in the final year age-specific vectors of maturity, selectivity, stock and catch weights, were used in combination with the true values of steepness (either \(h=0.88\) or \(h=0.60\)) and
unexploited recruitment levels (R₀ in Beverton Holt model) to estimate MSY associated biological reference points (SSB_{MSY}, F_{MSY}, and MSY). Calculations were forced to mimic those in the stock assessment models, namely that the weight in the plus group does not increase as older fish accumulate.

**Results**

Results for all simulations are presented in a standardized format to facilitate comparisons between models. For each case simulated, we calculated the relative error in the estimates of the two stock-recruit parameter (h, and R₀) and three reference points (MSY, SSB_{MSY}, and F_{MSY}), where relative error was calculated as (estimate-true value)/(true value). Relative errors are summarized over all 100 case-specific iterations by a box percentile plot, where the width at any point is proportional to the percentile. An unbiased estimate would be centered at 0 relative error, and a highly precise estimate would have very little vertical spread. Horizontal lines within the box percentile plot indicate the 25th-50th-75th percentiles (Figure 3). When comparing the box-percentile plots between the estimated quantities, we point out that the range of relative error is limited in the case of steepness because the true value has mathematical bounds of [0.2, 1.0]. That means that the maximum range of relative errors was constrained to the range [-0.773, 0.667].

**External estimates from VPA and ASAP**

VPA and ASAP estimates of SSB and recruitment from the 1977 model runs were passed to SRfit, where the stock recruit (S-R) relationship and MSY reference points were estimated. When there was no variability in recruitment, and true steepness was high (0.88), both models were unbiased and highly precise in estimating steepness (h) and virgin recruitment (R₀) in the Beverton-Holt model (Figure 4a). The pattern in bias and precision for the S-R parameters translated directly into the bias and precision in estimated biological reference points. SSB_{MSY} and MSY followed the same pattern as R₀ (Figure 4b) and F_{MSY} followed the same pattern as steepness (Figure 4c). This is not surprising, given that SSB_{MSY} and MSY are obtained by scaling SSB/R and YPR by the magnitude of recruitment at MSY, and F_{MSY} is directly related to a species resiliency, which is captured in the steepness parameter.

Estimates of steepness were invariant to the level of CV on recruitment, but estimates of R₀ were highly sensitive, especially in the VPA. With recruitment CV at 80%, the relative error in VPA estimates of R₀ were well over 1. In all cases, the bias on R₀, when present, was positive. The positive bias means that the estimates of MSY and SSB_{MSY} were overestimating the true values.

For cases where the true steepness was lower (0.6), the estimates of steepness were still very precise and fairly unbiased (Figure 5). However, estimates of R₀ were quite
imprecise even with no variability in recruitment (CV=0% cases). In addition, the VPA was negatively biased with no variability in recruitment, while ASAP was only slightly negatively biased (median was slightly < 0). At higher levels of recruitment variability, the imprecision grew as did the bias, although ASAP was always less biased than the VPA estimates. As before, the patterns in relative errors for $R_0$ and $h$ carried through to MSY, SSB$_{MSY}$ and F$_{MSY}$.

The general conclusion between externally estimated BRPs from VPA and ASAP runs on the same data sets is that the bias in estimates of the stock-recruit parameters was similar, but slightly less for ASAP, which carried through to less bias in the BRPs.

**Internal versus external estimates**

The internal S-R and BRP estimates from the ASAP 1977 model runs were compared to the externally estimated values from SRFit. As was the case with the external estimates (described above), the internal estimates of steepness were very accurate and precise, and this result was invariant to the level of recruitment CV and the true steepness value considered (Figures 6, 7). The precision of $R_0$ estimates was similar between internal and external estimates for the high steepness case ($h=0.88$), however, the bias pattern was inconsistent. With no recruitment variability, the external estimates had no bias, while the internal estimates were slightly positively biased. This may have been caused by misspecification within ASAP, which was set up to estimate non-zero recruitment deviations. With recruitment CV at 20% or 80%, the external estimates of $R_0$ had increasingly positive bias, while the internal estimates of $R_0$ were far more robust, with only slightly more positive bias compared to no recruitment variability. For the low steepness case ($h=0.6$), the loss of precision and increase in bias was severe for the external estimates of $R_0$, whereas the bias and precision of internal estimates were very similar between the two steepness levels. Again, the bias in $R_0$ estimates was always positive, which produced positive bias in MSY and SSB$_{MSY}$. The bias and precision in the BRPs followed exactly the pattern in the S-R parameters, as discussed above.

The general conclusion between internally versus externally estimated S-R parameters and BRPs from ASAP is that the external estimates of $R_0$ were generally less precise, but slightly less biased for CV=0% and CV=20%. However, the bias in external estimates was quite severe when CV=80%. There was no real difference in steepness estimates, except for the case with recruitment CV=80%, where the external estimates were biased slightly high. One possible source for this pattern of results is in the specification of recruitment variability between ASAP and SRFit. Further exploration of these simulated data sets might reveal whether a sensible default is to assume either greater or lesser variability. This next step would require a lot more hands on tuning than was possible for this meeting.

**Effect of extending the time series on ASAP estimates**

The pattern of results for S-R and BRP estimates was similar for model runs on the three time periods of data (1935, 1963, and 1977) whether estimated internally or externally.
For the high steepness cases (Figure 10), extending the time series to earlier years provided no gain in precision or bias in the estimates of steepness because they were already extremely precise and unbiased. There is a gain for estimates of R0 for runs beginning in 1963 rather than 1977 as there is less bias and somewhat more precision. However, there is no gain in extending the data back to 1935 as the bias is generally of the same magnitude as the 1977 runs.

For the low steepness cases (Figure 9), extending the time series further back in time led to less precision and far more bias. Runs from 1935 were always very biased, often settling at or near the upper bound of the steepness constraint (also true of 1963 run with CV=0). The steepness boundary solutions resulted in negative bias in R0 estimates for the same cases (1935 runs, and 1963 run with CV=0). Comparing 1963 runs to 1977 runs, steepness is slightly positively biased in the 1963 runs whereas it is unbiased in the 1977 runs. For R0, the 1963 runs with CV>0 were more precise and less biased than the 1977 runs.

The general conclusion from comparing ASAP assessments on the three time periods of data is that the model performance was improved by extending the series as far back as there were indices (1963), but extending back to 1935 when only total catch was available produced no gain and oftentimes exacerbated the bias. The fact that precision was better in those 1935 runs, even though they could be more biased, is further cause for caution. In an actual stock assessment, only the improved precision—not the bias—would be seen and thus, one could erroneously conclude that including historical total landings without indices was an improved model.

**Evaluating the use of short time series to estimate BRPs**

The VPA 1977 runs were compared with VPA 1995 runs to evaluate the ability of a short time series (1995-2006) to provide informative SSB and recruitment series from which to estimate S-R parameters and BRPs (using SRFit). For the high steepness cases (Figure 10), estimates of steepness from 1977 model runs were very precise and unbiased at all levels of recruitment CV, but for the 1995 runs, the steepness estimate started out unbiased (CV=0%) but the bias became negative as recruitment deviations increased. Virgin recruitment estimates were unbiased with no recruitment CV, but as that increased, the 1995 runs had enormous bias (>> 100%).

For the low steepness cases (Figure 11), the steepness estimates for both time periods had a slight positive bias with no recruitment deviations. However, with CV>0%, steepness estimates were mostly unbiased and very precise. Estimates of R0 were very poorly estimated in both time periods. With no recruitment CV, both VPA runs had strong negative bias, but the 1995 run was also very imprecise. With CV>0%, the bias became increasingly very positive, and precision was unacceptably poor for both VPA time series.

The general conclusion from comparing the two VPA runs using catch at age data from 1977 versus 1995 is that the shorter time series (only 12 years of data) did a very poor job of estimating unexploited levels of recruitment. This could be due to the length of the
time series, or to the limited amount of contrast in stock size (the depletion level in SSB was pretty flat over that time period, ranging from 6% to 16%). It is likely a combination of those two reasons (and perhaps others, as well). This conclusion comes from comparing the CV=0% cases between the high and low steepness model runs. With a true steepness of 0.88, when SSB is depleted to 20% of unexploited levels, $R_0$ is only decreased by 12%. Thus, even though the stock was very depleted for both the 1977 and 1995 runs, the recruitment levels were not nearly as depleted. In contrast, when the true steepness was 0.6, both SSB and recruitment were well below the unexploited levels, and consequently the bias in estimates of $R_0$ was negative. As recruitment CV increased, more random deviates in the upper tail of the assumed lognormal distribution would have been generated, and consequently the bias in $R_0$ became positive. This was still a problem even with the high steepness case, especially when recruitment CV=80%. Based on the cases explored in this simulation, we don’t believe that short time series from an overfished stock are likely to produce informative time series of SSB and recruitment from which to estimate BRPs.

Conclusions

Between externally estimated BRPs from VPA and ASAP runs on the same data sets, the bias in estimates of the stock-recruit parameters was similar, but slightly less for ASAP, which carried through to less bias in the BRPs. Comparing internally versus externally estimated stock-recruit parameters for ASAP, the external estimates of $R_0$ were generally less precise, but slightly less biased for CV=0% and CV=20%. However, the bias in external estimates was quite severe when CV=80%. This may relate to misspecification in the default level of recruitment variability assumed in ASAP and SRFit; it would require further detailed tuning to evaluate the impact of that model setting. When the ASAP model was applied to data from three different time periods with different amounts of data in each period, we found that the model performance was improved by extending the series as far back as there were indices (1963), but extending back to 1935 when only total catch was available produced no gain and oftentimes exacerbated the bias. For the VPA model runs using catch at age data that started in 1977 or in 1995, the shorter time series (only 12 years of data) did a very poor job of estimating unexploited levels of recruitment. This could be due to the length of the time series, or to the limited amount of contrast in stock size (the depletion level in SSB was pretty flat over that time period, ranging from 6% to 16%). Although there is not time to fully evaluate these hypotheses, based on the cases explored in this simulation, we conclude that short time series from an overfished stock are likely to produce informative time series of SSB and recruitment from which to estimate BRPs.
Table 1. Summary of basic dimensions along with biological and fishery characteristics of the simulated stock in PopSim.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of years</td>
<td>72</td>
</tr>
<tr>
<td>number of ages</td>
<td>15</td>
</tr>
<tr>
<td>plus group age</td>
<td>6</td>
</tr>
<tr>
<td>number of lengths</td>
<td>70</td>
</tr>
<tr>
<td>number market categories</td>
<td>2</td>
</tr>
<tr>
<td>number surveys</td>
<td>2</td>
</tr>
<tr>
<td>CV of surveys</td>
<td>0.5, 0.4</td>
</tr>
<tr>
<td>CV for recruitment</td>
<td>0.0001</td>
</tr>
<tr>
<td>CV on catch in weight</td>
<td>0.01</td>
</tr>
<tr>
<td>Growth stddevs</td>
<td>0.2</td>
</tr>
<tr>
<td>von B Linf</td>
<td>50</td>
</tr>
<tr>
<td>von B K</td>
<td>0.33</td>
</tr>
<tr>
<td>von B t0</td>
<td>-0.75</td>
</tr>
<tr>
<td>50% Mature (size)</td>
<td>30 cm</td>
</tr>
<tr>
<td>50% Selectivity Flat-topped</td>
<td>29 cm</td>
</tr>
<tr>
<td>M</td>
<td>0.2</td>
</tr>
</tbody>
</table>
Figure 1. Diagram showing the type of data available and the span of years where that information “exists” for use in the assessment models.
Figure 2. Fishing mortality rate multiplier (top) and resulting spawning stock biomass (SSB) for years 1935-2006.
Figure 3. Diagram of a box percentile plot, with horizontal lines indicating the 25th-50th-75th percentiles.
Figure 4a. Comparison of estimated stock recruitment parameters ($R_0$, unexploited recruitment, and $h$, steepness) between VPA and ASAP. These results are for the case where true steepness was 0.88 and the models started in 1977. Estimates were made in SRFit.
Figure 4b. Comparison of estimated MSY and SSB_{MSY} between VPA and ASAP. These results are for the case where true steepness was 0.88 and the models started in 1977. Estimates were made in SRFit.
Figure 4c. Comparison of estimated $F_{MSY}$ between VPA and ASAP. These results are for the case where true steepness was 0.88 and the models started in 1977. Estimates were made in SRFit.
Figure 5. Comparison of estimated stock recruitment parameters ($R_0$, unexploited recruitment, and $h$, steepness) between VPA and ASAP. These results are for the case where true steepness was 0.6 and the models started in 1977. Estimates were made in SRFit.
Figure 6. Comparison of internally (within ASAP) estimated stock recruitment parameters and those estimated externally in SRFit with ASAP estimates of SSB and recruitment. These results are for the case where true steepness was 0.88 and the models started in 1977.
Figure 7. Comparison of internally (within ASAP) estimated stock recruitment parameters and those estimated externally in SRFit with ASAP estimates of SSB and recruitment. These results are for the case where true steepness was 0.60 and the models started in 1977.
Figure 8a. Comparison of steepness estimated internally from three ASAP model runs starting in year 1935, 1963, or 1977. The data available in each period is summarized in Figure 1. True steepness was 0.88.
Figure 8b. Comparison of $R_0$ estimated internally from three ASAP model runs starting in year 1935, 1963, or 1977. The data available in each period is summarized in Figure 1. True steepness was 0.88.
Figure 9a. Comparison of steepness estimated internally from three ASAP model runs starting in year 1935, 1963, or 1977. The data available in each period is summarized in Figure 1. True steepness was 0.6.
Figure 9b. Comparison of $R_0$ estimated internally from three ASAP model runs starting in year 1935, 1963, or 1977. The data available in each period is summarized in Figure 1. True steepness was 0.6.
Figure 10a. Comparison of steepness estimated externally in SRFit from SSB and recruitment estimates corresponding to two VPA model runs starting in year 1977 and 1995. True steepness was 0.88.
Figure 10b. Comparison of $R_0$ estimated externally in SRFit from SSB and recruitment estimates corresponding to two VPA model runs starting in year 1977 and 1995. True steepness was 0.88.
Figure 11a. Comparison of steepness estimated externally in SRFit from SSB and recruitment estimates corresponding to two VPA model runs starting in year 1977 and 1995. True steepness was 0.6.
Figure 11b. Comparison of R₀ estimated externally in SRFit from SSB and recruitment estimates corresponding to two VPA model runs starting in year 1977 and 1995. True steepness was 0.6.