Adventures in Modeling Good and Bad Outputs: A 30 Year Retrospective

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- (Any errors, opinions, or conclusions are those of the author and should not be attributed to the U.S. Environmental Protection Agency)

Introduction

- Society has determined that undesirable by-products of production activities need to be regulated
  - Pollution abatement does not eliminate the undesirable byproduct of good output production
  - Pollution abatement transform bad output from one media (air) to another media (solid waste) where it constitutes a reduced threat to human health and the environment

Introduction (continued)

- Pollution Abatement (PA) and Productivity
  - PA divert inputs from good output production to use in PA
  - Difference in good output growth by regulated and unregulated technologies is change in traditional productivity associated with PA
  - Traditional productivity ignores output of PA - reduced bad output production
- Strategies for reducing bad outputs
  - Reduce good outputs
  - Reassigning inputs from producing good outputs to pollution abatement
  - Change quality of inputs

Introduction (continued)

- Benefits of environmental regulations accrue to society in form of improved quality of life
- Concerns remain about effect of regulations on economic health of economy
  - Are regulations optimal (i.e., MB = MC)?
  - Do regulations affect the "competitiveness" of an economy?
  - Traditional productivity ignores output of PA - reduced bad output production

Introduction (continued)

- Assigned input model (data requirements)
  - Good output production
  - Inputs assigned to good output production
  - Inputs assigned to pollution abatement
- Assigned input model (problems)
  - Survey of pollution abatement costs
  - Movement from end-of-pipe (EOP) to change-in process (CIP) abatement techniques increases difficulty of identifying inputs assigned to pollution abatement

Introduction (continued)

- Joint production model (data requirements)
  - Good output production
  - Bad output production
  - Exogenous inputs
- Joint production model (advantages)
  - No need to assign inputs to good output production and pollution abatement
- Previous surveys
  - Tyteca (1996)
  - Zhou, Ang, and Pho (2008)

Pre-DEA non-parametric models

- Early general equilibrium frameworks
  - Ayres and Kneese (1969)
  - Leontief (1970)
- Input-Output framework (shadow price of bad)
  - Lowe (1979)
- Activity analysis framework
  - Kohn (1975)
- CGE Models
  - Activity Analysis (Willett 1983)
  - Single activity (Smith and Espinosa 1996)

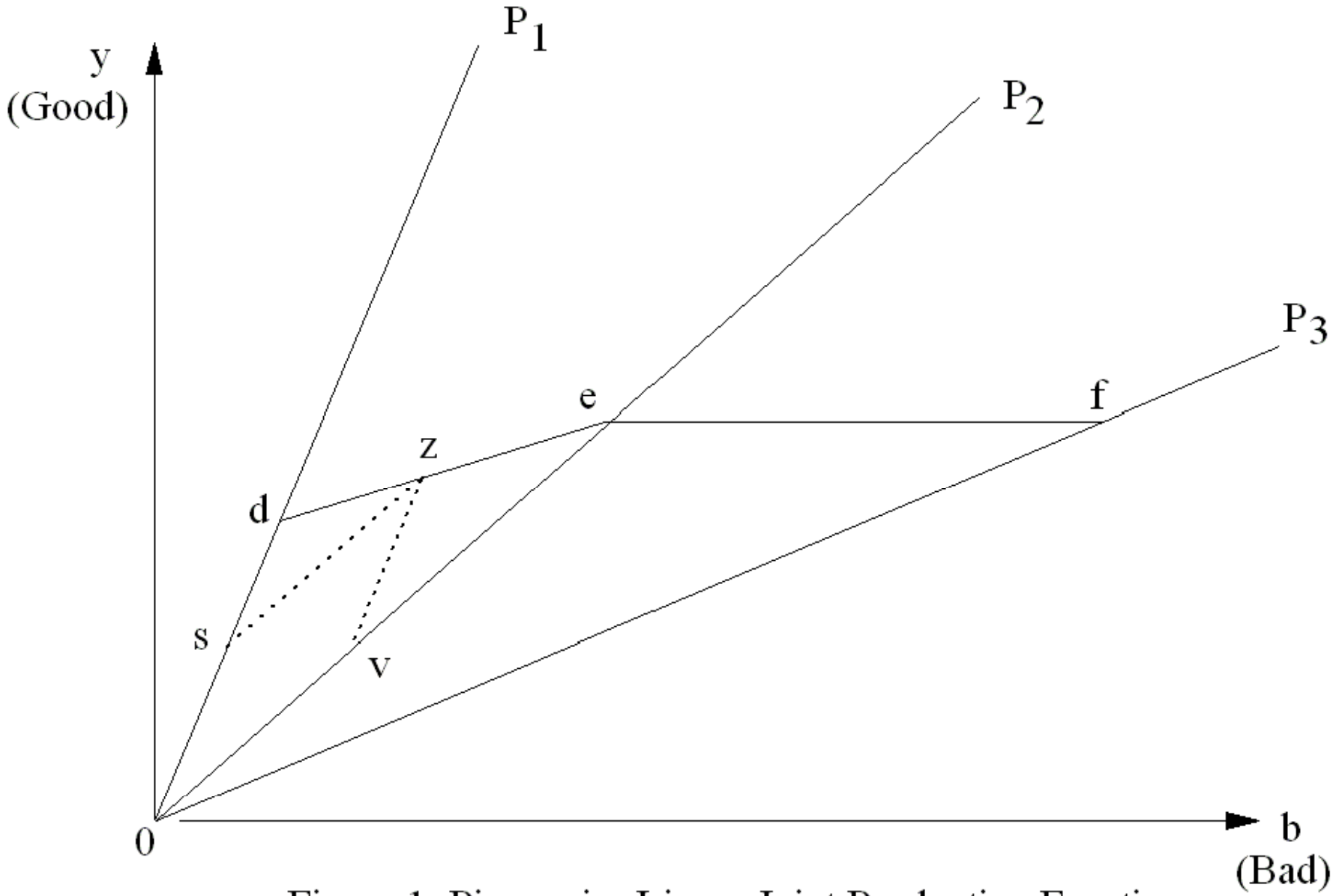

Figure 1. Piece-wise Linear Joint Production Frontier

DEA Non-parametric Models of Good and Bad Outputs

- Joint production models used to define PAC
  - Färe and Grosskopf (1983)
  - Färe, Grosskopf, and Pasurka (1986)
- Treat goods and bads symmetrically
- Technical efficiency involves expanding good and bad outputs
- Pollution abatement costs (PAC) is the opportunity cost of pollution abatement (foregone good output production)

Environmental Technology (regulated technology)

- Satisfy standard axioms
  - Inactivity is always possible
  - Finite inputs produce finite outputs
  - Inputs are freely disposable
- Environmental axioms
  - Weak disposability of outputs
  - Null jointness

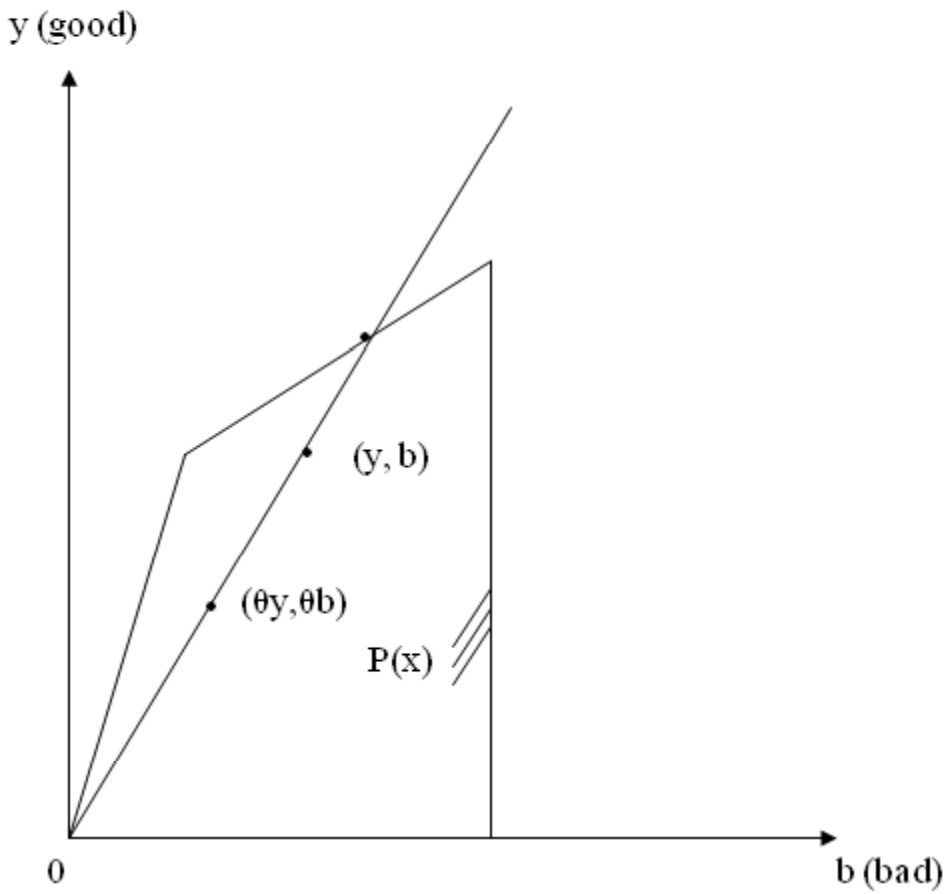

Figure 2. The Environmental Technology

PAC and Technical Efficiency

- Regulated technology – model bad output as not freely disposable
- Unregulated technology – model bad output as freely disposable
- PAC = good output production with unregulated frontier minus good output production with regulated frontier
- Advantages
  - Requires information on observed inputs and production of good and bad outputs
  - Requires no explicit information on PA activities
  - Capture synergies of abating emissions of two

PAC and Technical Efficiency (continued)

- Piecewise linear specification
  - Regulated and unregulated technologies
- Regulated technology (0EBC0)
  - Goods and bads are weakly disposable
- Unregulated (or least regulated) technology (0ABCD0)
  - Goods are freely disposable

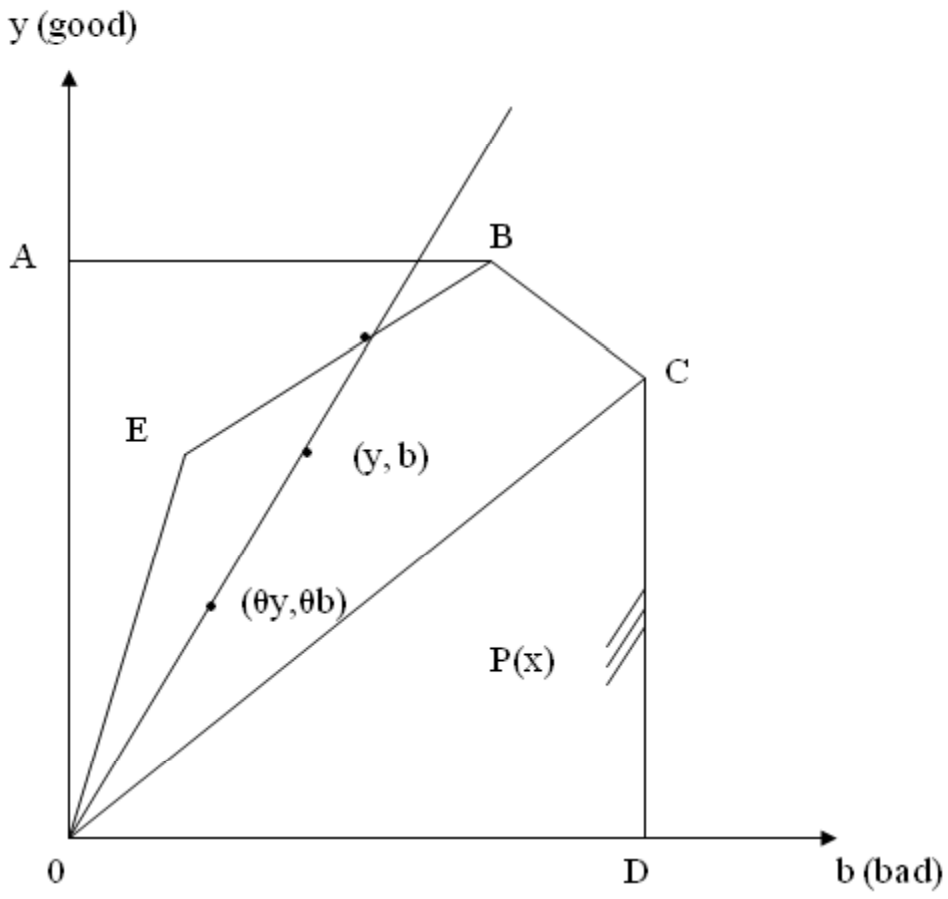

Figure 3. Färe, et al. (1986) frontier

$$\left(D_O^t(x^{t,k'}, y^{t,k'}, b^{t,k'})\right)^{-1} = \max \quad \beta \tag{4}$$

$$\begin{aligned}
s.t. \quad & \sum_{k=1}^{K} z_k^t y_{km} \geq (\beta) y_{k'm}^t, && m = 1,\ldots,M \\
& \sum_{k=1}^{K} z_k^t b_{kj} = (\beta) b_{k'j}^t, && j = 1,\ldots,J \\
& \sum_{k=1}^{K} z_k^t x_{kn} \leq x_{k'n}^t, && n = 1,\ldots,N \\
& z_k^t \geq 0, && k = 1,\ldots,K
\end{aligned}$$

PAC and Adjusted Technical Efficiency

- Färe, Grosskopf, Lovell, Pasurka (1989)
- Hyperbolic function
- Treat goods and bads asymmetrically
  - Technical efficiency - credit DMU for expanding good output production and contracting bad output production
- Joint production model also used to define PAC
- Regulated technology (0EBCD0) - NEW
- Unregulated technology (0ABCD0)

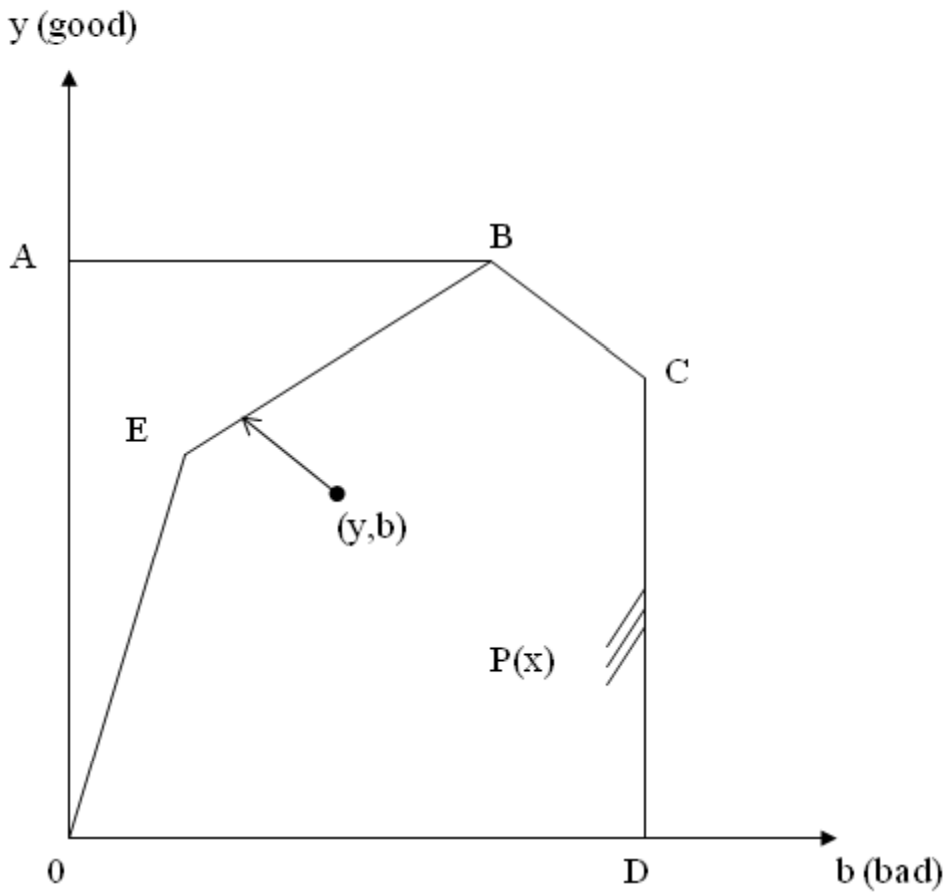

Figure 4. Färe, et al. (1989) frontier

$$
\begin{aligned}
H_O^t(x^{t,k'}, y^{t,k'}, b^{t,k'}) = \max \quad & \lambda \\
s.t. \quad & \sum_{k=1}^{K} z_k^t y_{km} \geq (\lambda) y_{k'm}^t, \qquad m = 1,\ldots,M \\
& \sum_{k=1}^{K} z_k^t b_{kj} = (1/\lambda) b_{k'j}^t, \qquad j = 1,\ldots,J \\
& \sum_{k=1}^{K} z_k^t x_{kn} \leq x_{k'n}^t, \qquad n = 1,\ldots,N \\
& z_k^t \geq 0, \qquad k = 1,\ldots,K
\end{aligned}
\tag{5}
$$

PAC and Adjusted Technical Efficiency (continued)

- Färe, Grosskopf, Lovell, Pasurka (1989)
- Hyperbolic function
- Treat goods and bads asymmetrically
- Propose a new specification of the unregulated technology
- Regulated technology (0EBCD0)
- Unregulated technology (0AFD0) - NEW

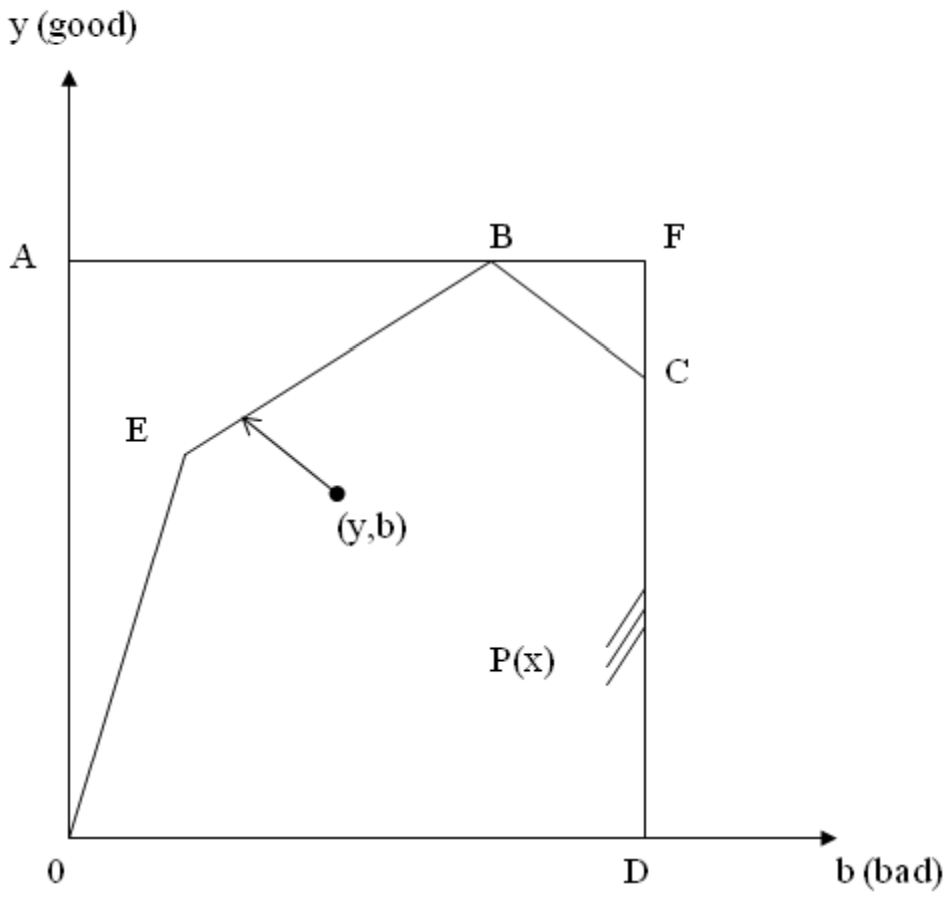

Figure 4. Färe, et al. (1989) frontier

Adjusted Productivity Change

- Directional distance function
  - Specify piecewise linear technologies
  - Problem – potential for infeasible LP problem with technology from period t and observation from period t+1
- Decompose productivity change into two components
  - Technical change – shifts in the production frontier
  - Changes in technical efficiency – change in distance of observation from maximum potential good output production

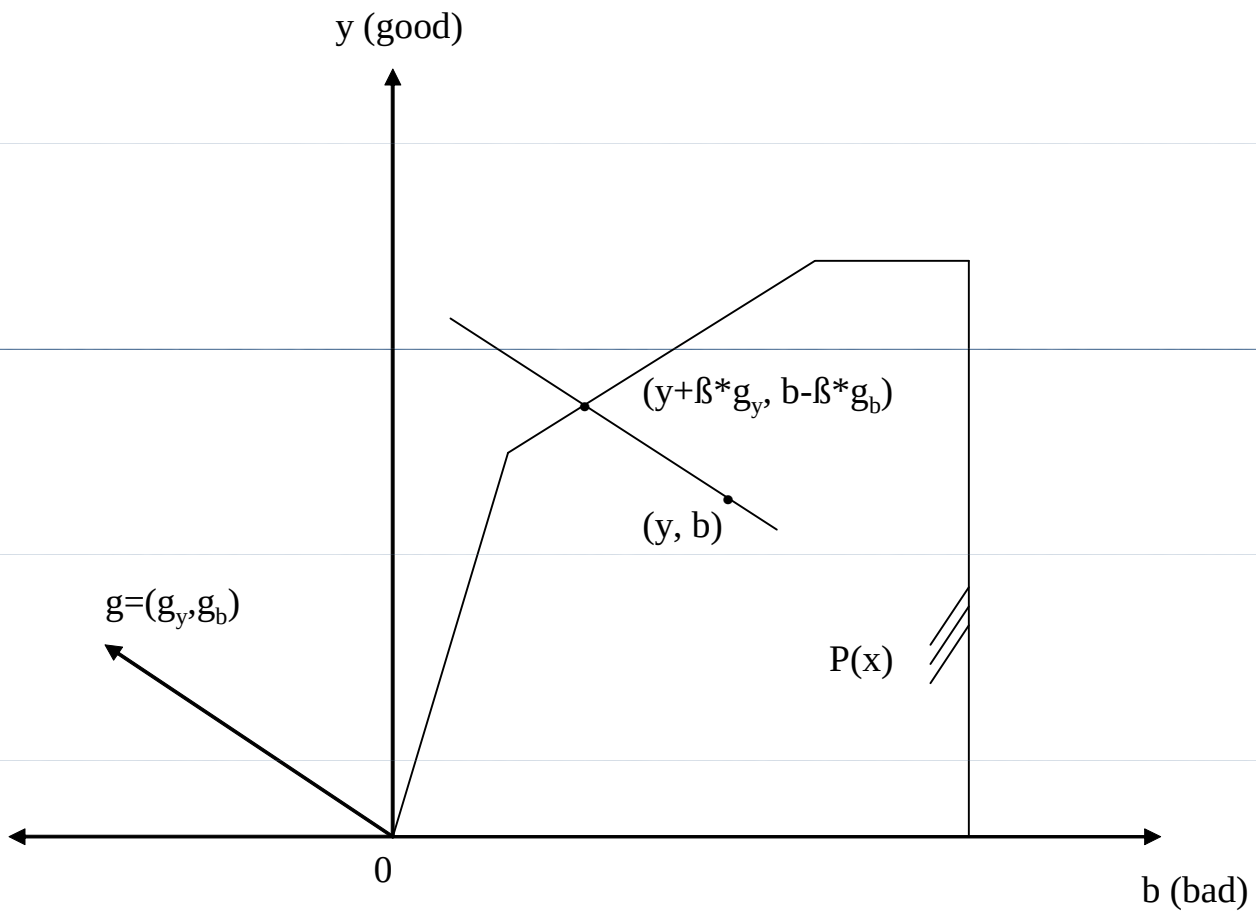

Figure 3. The Directional Output Distance Function

$$\vec{D}_O^t(x^{t,k'}, y^{t,k'}, b^{t,k'}; y^{t,k'}, -b^{t,k'}) = \max \quad \beta \tag{6}$$

$$s.t. \quad \sum_{k=1}^{K} z_k^t y_{km} \geq (1+\beta) y_{k'm}^t, \qquad m = 1,\ldots,M$$

$$\sum_{k=1}^{K} z_k^t b_{kj} = (1-\beta) b_{k'j}^t, \qquad j = 1,\ldots,J$$

$$\sum_{k=1}^{K} z_k^t x_{kn} \leq x_{k'n}^t, \qquad n = 1,\ldots,N$$

$$z_k^t \geq 0, \qquad k = 1,\ldots,K$$

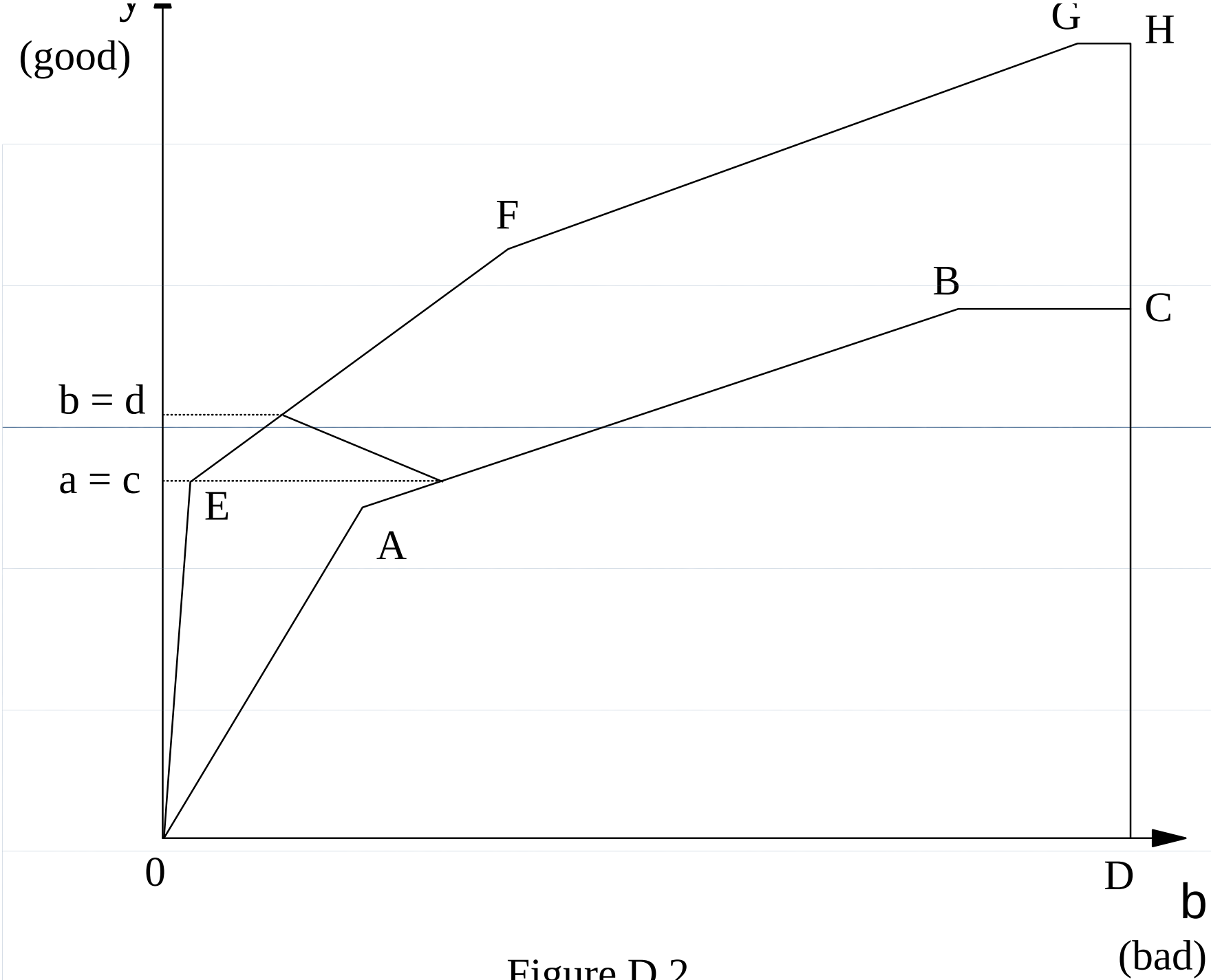

Figure D.2

Productivity Change

$$ML^{t+1} = \frac{\left[1+\vec{D}_o^{t+1}\left(x^t, y^t, b^t; y^t, -b^t\right)\right]}{\left[1+\vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)\right]}$$

$$ML_t^{t+1} = MLTECH_t^{t+1} \times MLEFFCH_t^{t+1}$$

Productivity Change (continued)

$$MLTECH_t^{t+1} = \left\{ \frac{\left[1 + \vec{D}_o^{t+1}\left(x^t, y^t, b^t; y^t, -b^t\right)\right]}{\left[1 + \vec{D}_o^{t}\left(x^t, y^t, b^t; y^t, -b^t\right)\right]} \right\}$$

and

$$MLEFFCH_t^{t+1} = \frac{\left[1 + \vec{D}_o^{t}\left(x^t, y^t, b^t; y^t, -b^t\right)\right]}{\left[1 + \vec{D}_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)\right]}$$

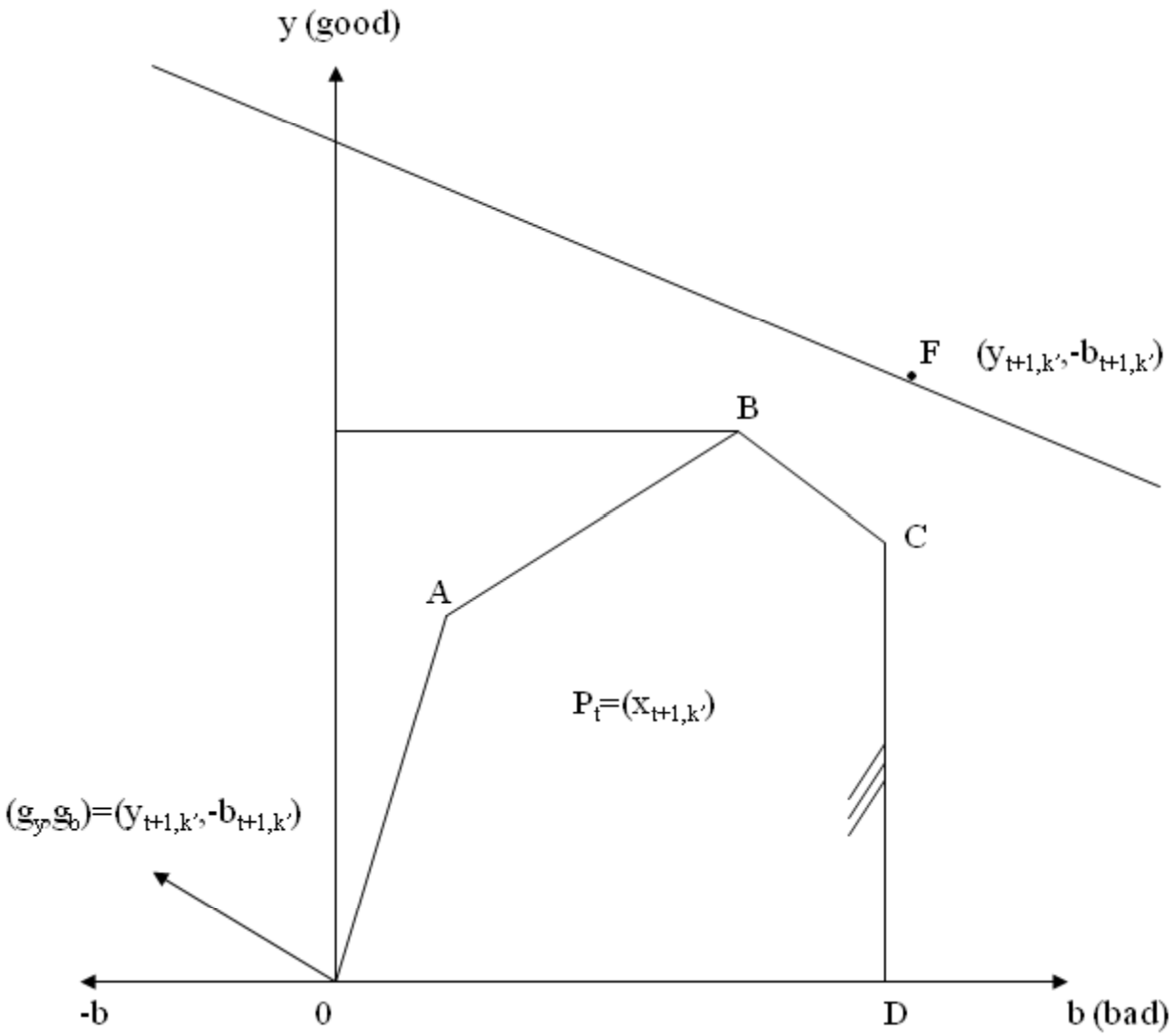

Figure 6. No Solution Example

PA and Traditional Productivity

- Traditional productivity
  - Specify piecewise linear regulated and unregulated environmental production functions
  - Hold bad output constant and scale on good output
  - Environment production function is special case of environmental directional distance function (Färe, Grosskopf, and Pasurka (2007a)
  - Investigate relative importance of PA on traditional productivity by calculating productivity change for regulated and unregulated frontiers (Färe, Grosskopf, and Pasurka 2007b)

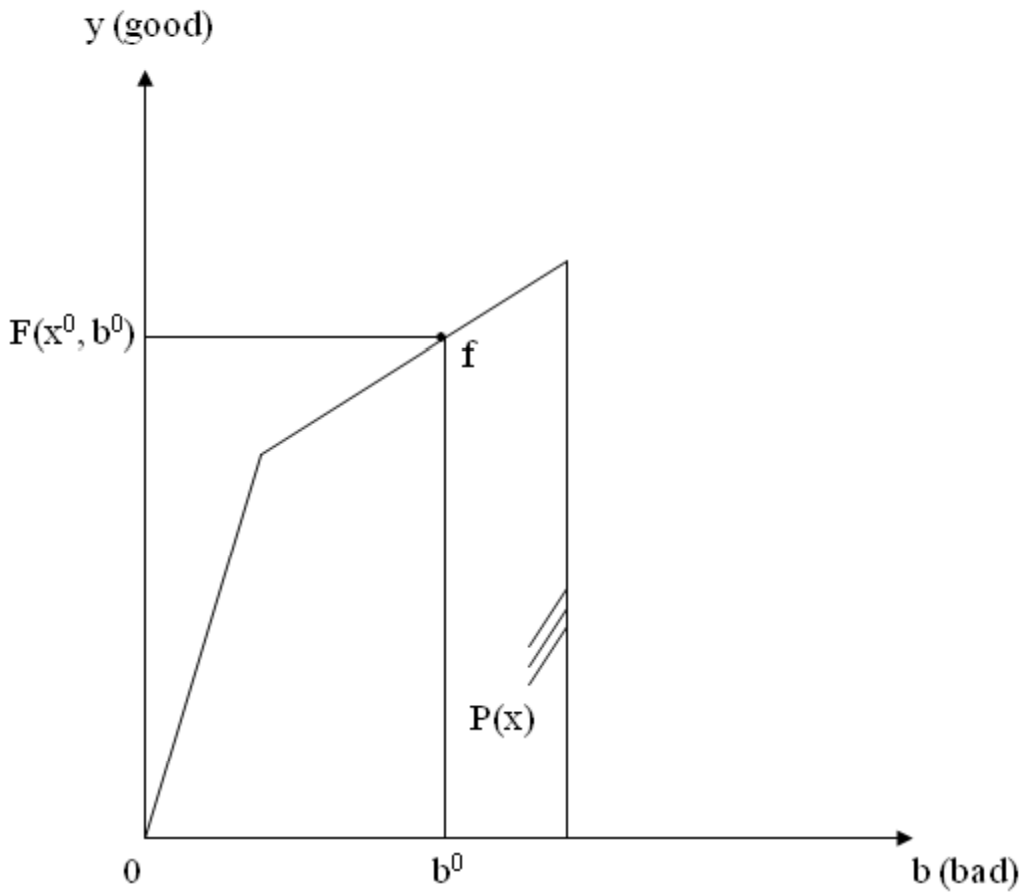

Figure 7. The Environmental Production Function

$$\vec{D}_o(x^{k'}, y^{k'}, b^{k'}; g_y, g_b) = \max \beta^{k'} \tag{7}$$

$$s.t. \quad \sum_{k=1}^{K} z_k y_{km} \geq y_{k'm} + \beta^{k'} g_{y_m}, \qquad m = 1,\ldots,M$$

$$\sum_{k=1}^{K} z_k b_{kj} = b_{k'j} - \beta^{k'} g_{b_j}, \qquad j = 1,\ldots,J$$

$$\sum_{k=1}^{K} z_k x_{kn} \leq x_{k'n}, \qquad n = 1,\ldots,N$$

$$z_k \geq 0 \qquad k = 1,\ldots,K$$

$$F(x^{k'}, b^{k'}) = \max \sum_{k=1}^{K} z_k y_k \tag{8}$$

$$s.t. \quad \sum_{k=1}^{K} z_k b_{kj} = b_{k'j}, \qquad j = 1, \ldots, J$$

$$\sum_{k=1}^{K} z_k x_{kn} \le x_{k'n}, \qquad n = 1, \ldots, N$$

$$z_k^t \ge 0, \qquad k = 1, \ldots, K$$

Tradable Permits

- Brännlund, et al. (1998)
- Färe, Grosskopf, Pasurka (mimeo)
  - Environmental production function
  - Propose a new specification of the regulated technology
  - Regulated technology (0EBFD0) - NEW
  - Unregulated technology (0AFD0)

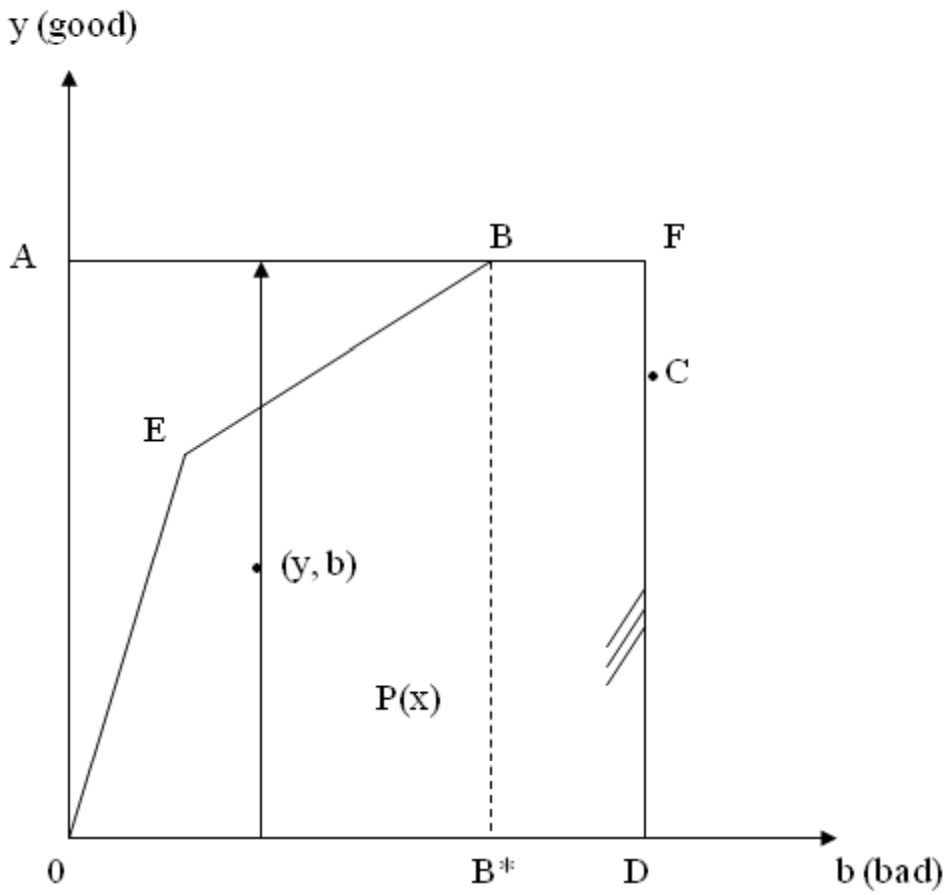

Figure 8. Färe, Grosskopf, Pasurka (2012a) frontier

More Applications of Joint Production Model

- Changes in PAC
  - Specify piecewise linear regulated and unregulated environmental production functions
  - Investigate relative importance of factors associated with changes in vertical distance between the regulated and unregulated frontiers
  - Pasurka (2001)
  - Färe, Grosskopf, and Pasurka (in progress)
- Changes in bad output production
  - Investigate relative importance of factors associated with changes in changes in bad output production
  - Attempt to provide more direct link between joint production models and existing input-output and divisia decomposition literature
  - Pasurka (2006)

Variations on DEA joint production model

- Environmental performance indexes (EPI) – ration of good to bad output
  - Färe, Grosskopf, and Hernando-Sanchez 2004
  - Färe, Grosskopf, and Pasurka 2006
  - Färe, Grosskopf, and Pasurka 2012 – multiple bad outputs

$$= \frac{y^{t+1} / y^{t}}{b^{t+1} / b^{t}} = \frac{y^{t+1} / b^{t+1}}{y^{t} / b^{t}} \tag{20}$$

$$D_u(b_{k'}^t)^{-1} = \min \lambda \tag{22}$$

$$s.t. \quad \sum_{t=1}^{T} \sum_{k=1}^{K} z_k^t b_{kj}^t \le \lambda b_{k'j}^t \qquad j = 1,\ldots, J$$

$$\sum_{t=1}^{T} \sum_{k=1}^{K} z_k^t = 1$$

$$z_k^t \ge 0, \quad k = 1,\ldots, K, \quad t = 1,..,T$$

$$= \frac{y^{t+1} / y^{t}}{D_b(b^{t+1}) / D_b(b^{t})} = \frac{y^{t+1} / D_b(b^{t+1})}{y^{t} / D_b(b^{t})} \tag{26}$$

Variations on DEA joint production model (continued)

- No data on PA inputs or bad outputs
  - Färe, Grosskopf, and Pasurka 1989
  - Belova, Morgan, Pasurka, and Shadbegian (in progress)

Network Model

- Regulated technology – inputs assigned to either good output production or PA (2 subtechnologies)
- Characteristics of network technology
  - Specify separate abatement process
  - Good output can serve as input in abatement process
  - Gross bad output production is input into abatement process
  - Reduced bad output production is output of abatement process
  - Net bad output production is output of production technology
- Unregulated technology – all inputs are available for good output production

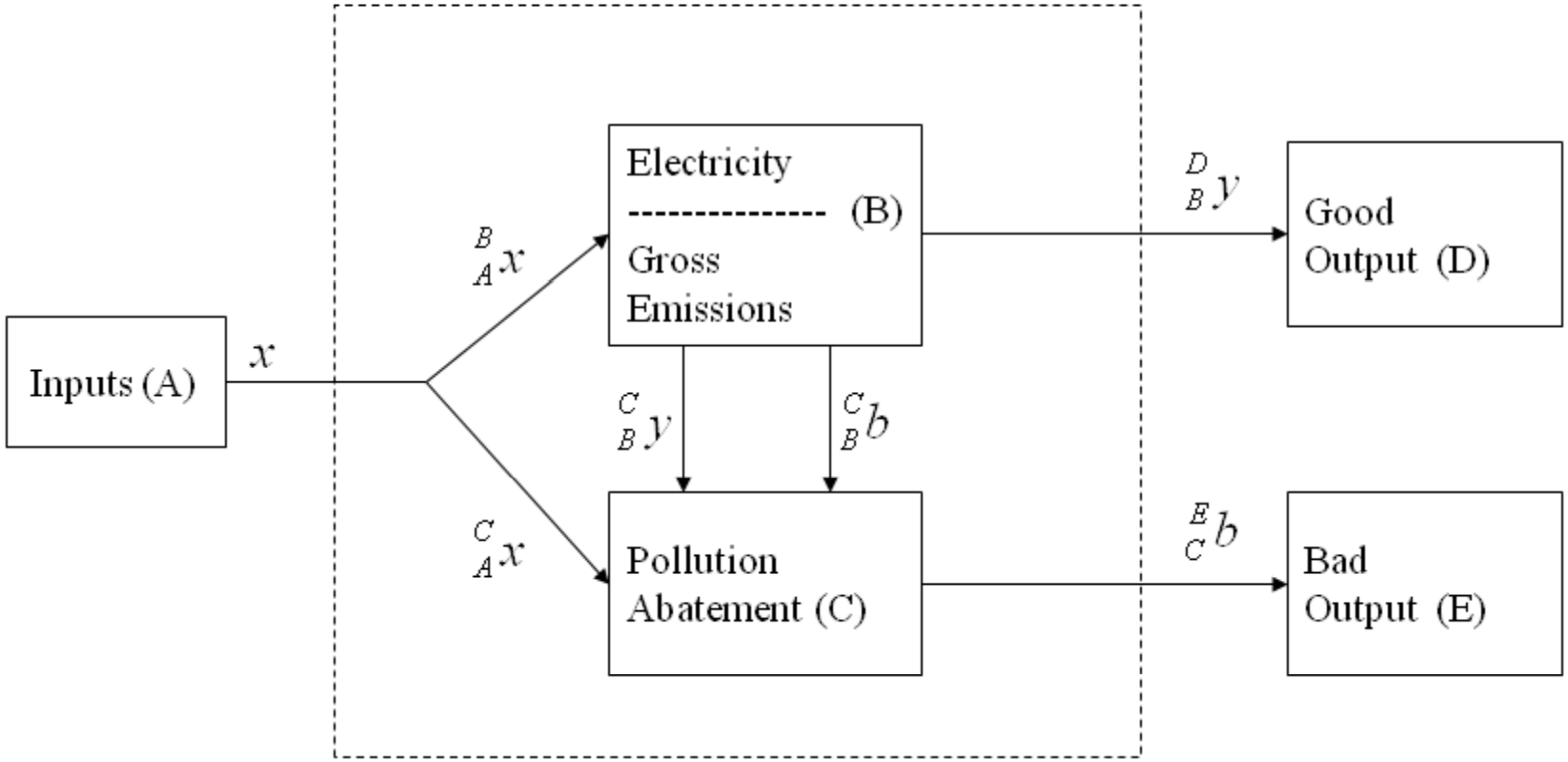

Figure 9. Network Technology of Subtechnologies

Input-Output DEA

- Input-Output DEA
  - tenRaa (1995) and tenRaa and Mohnen (2002) - model good output production
  - Böhm and Luptáčik (2006 and 2010) – model good and bad output production
- DEA with multiple input-output tables to model an economy using a Network Model
  - Prieto and Zofío (2007) – technical efficiency of 5 OECD countries
  - Prieto (2001) – technical change with no empirical example (unpublished paper)

Input-Output DEA (continued)

- "Productivity Change in a Network Input-Output Model with Bad Outputs"
  - Pasurka (2012)
  - Exxtend Prieto and Zofío (2007) – model good and bad output production with input-output tables from multiple years

Parametric Joint Production Models

- Shadow prices of bad outputs
  - Translog distance function (Färe, Grosskopf, Lovell, and Yaisawarng 1993)
  - Quadratic directional distance function (Färe, Grosskopf, Noh, and Weber 2005)
  - Comparison of effect of different scaling directions (Vardanyan and Noh 2006)
  - Comparison of translog distance function and quadratic directional distance function (Färe, Martins-Filho, and Noh 2010)
- Additional applications
  - Substitutability between good output and bad output (Färe, Grosskopf, Noh, and Weber 2005)
  - Substitutability and complementary among bad outputs (Färe, Grosskopf, Pasurka, and Weber 2012)

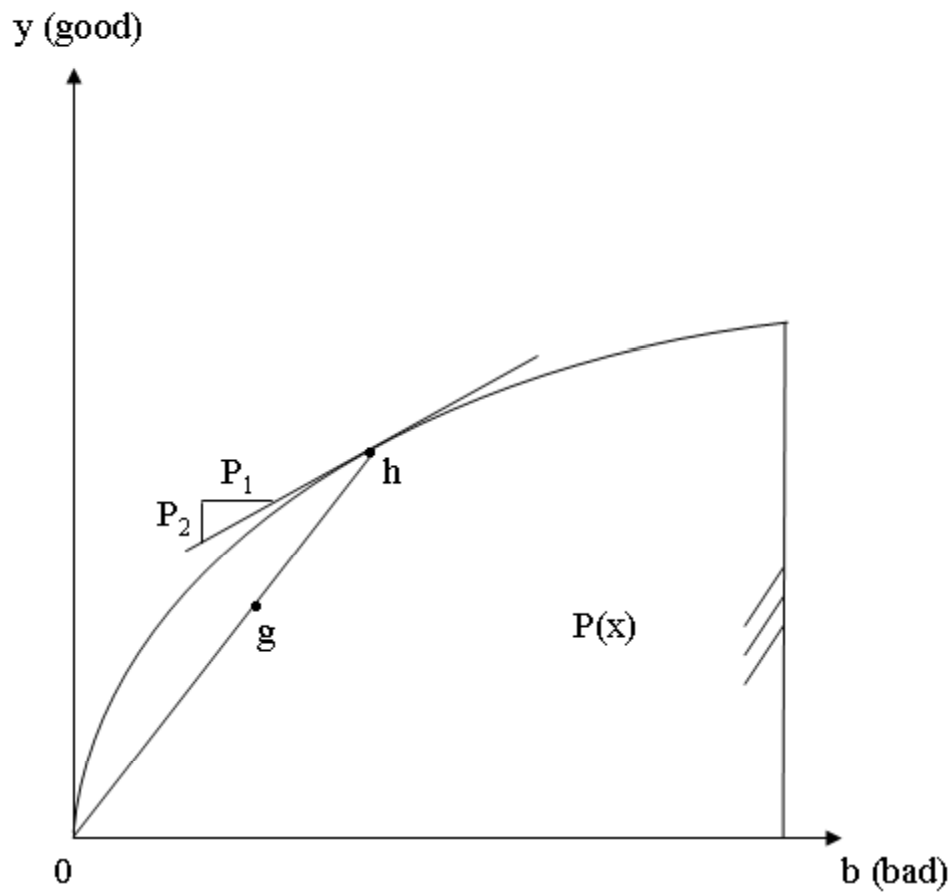

Figure 10. Parametric Joint Production Technology

$$\ln D_o(y_k, b_k, x_k) = \alpha_0 + \sum_{n=1}^{N} \beta_n \ln x_{nk} + \frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \beta_{nn'} \ln x_{nk} \ln x_{n'k} + \sum_{m=1}^{M} \nu_m \ln y_{mk} + \sum_{j=1}^{J} \gamma_i \ln b_{jk}$$

$$+ \frac{1}{2}\sum_{m=1}^{M}\sum_{m'=1}^{M} \nu_{mm'} \ln y_{mk} \ln y_{m'k} + \sum_{m=1}^{M}\sum_{j=1}^{J} \delta_{mj} \ln y_{mk} \ln b_{jk} + \frac{1}{2}\sum_{j=1}^{J}\sum_{j'=1}^{J} \gamma_{jj'} \ln b_{jk} \ln b_{j'k} + \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{nm} \ln x_{nk} \ln y_{mk} \tag{6}$$

$$+ \frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{J} \varphi_{nj} \ln x_{nk} \ln b_{jk}$$

$$\vec{D}_o(y_k, b_k, x_k; 1, -1) = \alpha_0 + \sum_{n=1}^{N} \beta_n x_{nk} + \frac{1}{2}\sum_{n=1}^{N}\sum_{n'=1}^{N} \beta_{nn'} x_{nk} x_{n'k} + \sum_{m=1}^{M} \nu_m y_{mk} + \sum_{j=1}^{J} \gamma_j b_{jk}$$

$$+ \frac{1}{2}\sum_{m=1}^{M}\sum_{m'=1}^{M} \nu_{mm'} y_{mk} y_{m'k} + \sum_{m=1}^{M}\sum_{j=1}^{J} \delta_{mj} y_{mk} b_{jk} + \frac{1}{2}\sum_{j=1}^{J}\sum_{j'=1}^{J} \gamma_{jj'} b_{jk} b_{j'k} + \frac{1}{2}\sum_{n=1}^{N}\sum_{m=1}^{M} \psi_{nm} x_{nk} y_{mk}$$

$$+ \frac{1}{2}\sum_{n=1}^{N}\sum_{j=1}^{J} \varphi_{nj} x_{nk} b_{jk}$$

Cost Functions with Bad Outputs

- Tran and Smith (1983)
- Gollop and Roberts (1983, 1985)
- McClelland and Horowitz (1999)
- Ball, Färe, Grosskopf, and Zaim (2005)
  - Non-parametric cost function
  - Only cost function to impose null-jointness and weak disposability
- Chapple, Paul, and Harris (2005, 2006)
- Mosheim (2006)

Challenges

- Bads - are they inputs or outputs?
- Should we worry about hypothetical free disposability parallel universes?
- Material balances
  - RFF studies (conclude mid-1970s)
  - Coelli, Lauwers, and van Huylenbroeck (2007)
  - Førsund (2009)
  - Murty, Russell, and Levkoff (forthcoming),
- Data availability
  - Environmental Accounting
  - *Integrated Environmental and Economic Accounting for Fisheries* (2004)
- Peer acceptance

Challenges (continued)

- My perspective – to what extent can the joint production model provide a unified framework for addressing questions associated with pollution abatement?
- That's All Folks!!!
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