Deep sea red crab; Appendix 3

2-point boundary model

Estimation of Average Recruitment, Biomass Weighted F, and Equilibrium Catch

Two quantitative surveys of red crab abundance and long-term record of landings provide an opportunity to estimate the average recruitment necessary to support the observed time series of catch. This is accomplished by using a simple mass balance equation with boundary conditions defined as the initial and final survey values.

Process Equation

Let B_t represent the biomass at time t and specify the boundary conditions B_0 and B_T. The biomass at time t+1 can be expressed as

$$B_{t+1} = \left(B_t - C_t + R_t\right)S \qquad (1)$$

Where C_t is the total catch and R_t is total recruitment of biomass to the population. The parameter S can be thought of as either the survival rate = e^{-M} or the difference between the instantaneous rate of growth G and M or $S=e^{-(G-M)}$. For this application it was assumed that increments to population biomass via growth are included in the Rt term; therefore $S=e^{-M}$ No information is available to estimate the annual recruitment to the population but Eq. 1 can be simplified by let R_t equal a constant, say R.

$$B_{t+2} = \left(B_{t+1} - C_{t+1} + R\right)S \qquad (2)$$

Substituting Eq. 1 into 2 recursively leads to

$$B_{t+2} = \left(\left(B_t - C_t + R\right)S - C_{t+1} + R\right)S$$

$$B_{t+3} = \left(B_{t+2} - C_{t+2} + R\right)S$$

$$B_{t+3} = \left(\left(\left(B_t - C_t + R\right)S - C_{t+1} + R\right)S - C_{t+2} + R\right)S$$

….

$$B_{t+T} = B_t S^{T-1} + \sum_{j=1}^{T-1} S^j R - \sum_{j=1}^{T-1} C_j S^{T-j} \qquad (3)$$

If we let B_t=B(0), B_{t+T}=B(T) and assume S then it is possible to estimate R as the average recruitment necessary to satisfy Eq. 3.

$$R = \frac{B(T) - B(0)S^{T-1} + \sum_{j=1}^{T-1} C_j S^{T-j}}{\sum_{j=1}^{T-1} S^j} \qquad (4)$$

Given the average recruitment R, the year-specific F_t can be estimated as

$$\hat{F}_t \approx \frac{C_t}{B_t + R} \qquad (5)$$

The estimates of year specific F_t are unreliable since they depend on the average recruitment estimate R. However, the average F over the period can be estimated as

$$\overline{F} = \sum_{j=1}^{T-1} \frac{\widehat{F}_j}{T-1} \qquad (6)$$

The average catch sufficient to maintain the population at its current size can be estimated by setting B_{T+1}=B_T in Eq. 1 and solving for C as

$$\begin{aligned} B_T &= \left(B_T - \overline{C}_{EQ} + R\right)S \\ \overline{C}_{EQ} &= R - \frac{B_T(1-S)}{S} \end{aligned} \qquad (7)$$

Eq. 4, 6 and 7 can now be used to estimate the average recruitment necessary to support the total removals between time t and t+T, the average biomass weighted F experienced by the population, and the average catch necessary to maintain the population at its current value of B_T.

Incorporating the Uncertainty in Population Size

The uncertainty in initial and final population sizes has important implications for the uncertainty in the average R, Fbar and C_{EQ}. This uncertainty can be approximated by convolving the distribution of initial population size with the final population size. Assume that the survey mean estimates are normally distributed. Let $B_t \sim N(\mu_t\ \sigma_t^2)$, $B_{t+T} \sim N(\mu_{t+T}\ \sigma_{t+T}^2)$ and $\Phi(.)$ define the cdf of the normal distribution. The inverse of the normal cdf, say $\Phi^{-1}(.)$, can be used to define population estimates for equal probability intervals

$$\begin{aligned} B_{t,\alpha} &= \Phi^{-1}\left(\mu_t, \sigma_t^2, \alpha\right), \quad \alpha = \alpha_{\min}, \ldots \alpha_{\max} \\ B_{T,\beta} &= \Phi^{-1}\left(\mu_T, \sigma_T^2, \beta\right), \quad \beta = \beta_{\min}, \ldots \beta_{\max} \end{aligned} \qquad (8)$$

Define $R_{\alpha,\beta}$ as the average recruitment obtained by substituting $B_{t,\alpha}$ and $B_{T,\beta}$ in Eq. 4 for B(0) and B(T) respectively. The sampling distribution of R and by extension, Fbar and Cbar, can now be obtained by simply matching all possible values of α with all possible values of β. More economically, one can define a small step size, say δ and evaluate $R_{\alpha,\beta}$ for equal increments between the minimum and maximum values of the cdf. The sampling distribution of R, Fbar, and Ceq is just the collection of discrete estimates since all estimates $R_{\alpha,\beta}$ have equal probabilities of occurrence = δ^2 and the sum of all δ^2 's is one.

Application to Red Crab

Estimates of R, Fbar, and C_{EQ} were derived for male and female red crab from the 1974 and 2004 fishery independent surveys (Table A3-1) and landings from 1974 to 2003 (Table A3-2). The distributions of R, Fbar and CEQ were based on convolution of 51 equal probability cut points representing a 95% confidence interval for the initial and final year biomass estimates. The convolution distribution was based on 2601 (i.e. 51 x 51) evaluations of Eq. 4. Annual survival for the base runs was assumed to be 0.86 (i.e., M=0.15)

Model results suggest that the median male recruitment is about 8500 mt per year. Historical average F between 1974 and 2004 was about 0.04 (Table A3-3). Given the population size in 2004, catches of 2,060 mt would keep the population at its current size of about 36,000 mt. This is about 16% higher than the average catch between 1973 and 2007 but 10% less than landings since 2000.

Between 1974 and 2004 the female population (>90 mm CW) increased nearly four-fold from 15 kt to 55 kt. Under the assumption that fishing mortality on the females was essentially zero, the estimated median recruitment was 9837 mt. The confidence intervals for median recruitment levels for males and females overlap which suggest comparable rates of biomass recruitment. The parameters for average recruitment and survival are confounded and the small differences in average recruitment estimates between male and female recruitment could be due to slightly different mortality rates or growth rates between sexes. For example, assuming an M=0.13 for females results in a median R of 7,810 mt that is about the same as the median R for males when M=0.15.

The sensitivity of the R, Fbar and C_{EQ} to changes in M are illustrated in Tables A3-4 to A3-6. Estimated average recruitment increases about three-fold as M increases (or S declines) from 0.05 to 0.20. The estimated equilibrium catch is relatively unchanged remaining at about 2,000 mt. Figures A3-1 and A3-2 demonstrate that as S approaches 1 the long-term catch equals the estimated average recruitment.

Table A3-1. Estimated survey biomass of male and female red crab, 1974 and 2004.

Category	Initial Biomass (SE)	Final Biomass (SE)
Fishable Biomass of Males	30,302 (6,363)	36,247 (4,612)
Female Biomass (>90 mm CW)	15,654 (3,719)	55,279 (7,033)

Table A3-2. Summary of annual landings (mt) of red crab in US.

Year	Landings (mt)
73	112.5
74	503.1
75	307.3
76	637.9
77	1244.6
78	1247.6
79	1210.8
80	2481.2
81	3031.8
82	2445.6
83	3252.4
84	3875.0
85	2236.7
86	1248.7
87	2110.3
88	3592.7
89	2393.2
90	1526.7
91	1791.0
92	1061.2
93	1439.9
94	0.3
95	572.0
96	465.6
97	1725.2
98	1501.1
99	1869.2
00	3129.4
01	4002.7
02	2142.5
03	1920.0
04	2040.3
05	2013.2
06	1716.0
07	1284.0

Table A3-3. Estimated median recruitment, average F, and equilibrium catch based on 2-point boundary value method. Values in parentheses represent 90% confidence interval. Natural mortality is assumed to be 0.15 (S=0.861).

Category	Recruitment	Fishing Mortality	Equilibrium Catch
Fishable Biomass of Males	7,928 (6.856, 9,068)	0.042 (0.036, 0.049)	2,044 (2,023, 2,064)
Female Biomass (>90 mm CW)	9,044 (7,408, 10,785)	0	72 (52, 93)

Table A3-4. Estimated median recruitment, average F, and equilibrium catch based on 2-point boundary value method. Values in parentheses represent 90% confidence interval. Natural mortality is assumed to be 0.05 (S=0.95).

Category	Recruitment	Fishing Mortality	Equilibrium Catch
Fishable Biomass of Males	3,850 (3,402, 4,324)	0.047 (0.041, 0.054)	1,987 (1,819, 2,152)
Female Biomass (>90 mm CW)	3,427 (2,766, 4,127)	0	584 (419, 757)

Table A3-5. Estimated median recruitment, average F, and equilibrium catch based on 2-point boundary value method. Values in parentheses represent 90% confidence interval. Natural mortality is assumed to be 0.1 (S=0.905).

Category	Recruitment	Fishing Mortality	Equilibrium Catch
Fishable Biomass of Males	5,819 (5,095, 6587)	0.044 (0.038, 0.051)	1,996 (1,932, 2,058)
Female Biomass (>90 mm CW)	6,049 (4,945, 7,224)	0	219 (157, 283)

Table A3-6. Estimated median recruitment, average F, and equilibrium catch based on 2-point boundary value method. Values in parentheses represent 90% confidence interval. Natural mortality is assumed to be 0.2 (S=0.819).

Category	Recruitment	Fishing Mortality	Equilibrium Catch
Fishable Biomass of Males	10,159 (8,704, 11,707)	0.039 (0.034, 0.046)	2,110 (2,104, 2,116)
Female Biomass (>90 mm CW)	12,297 (10,077, 14,658)	0	22 (16, 28)

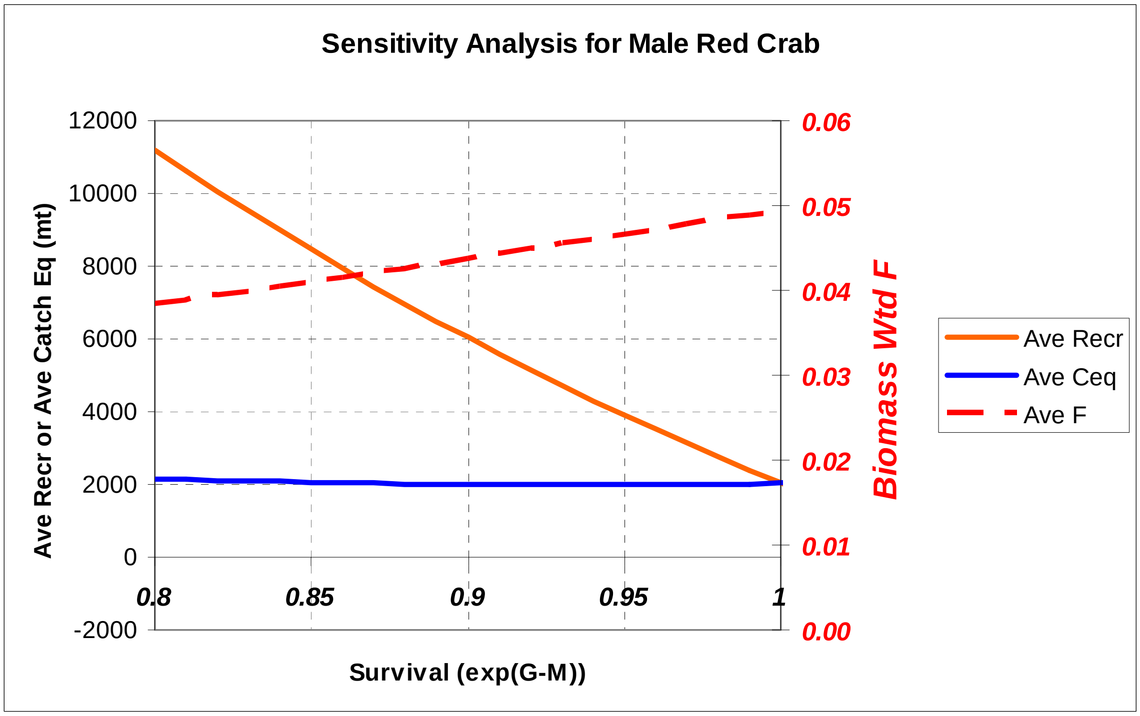

Fig A3-1. Sensitivity analysis of recruitment, average F and equilibrium catch for male red crab to varying levels of survival rate.

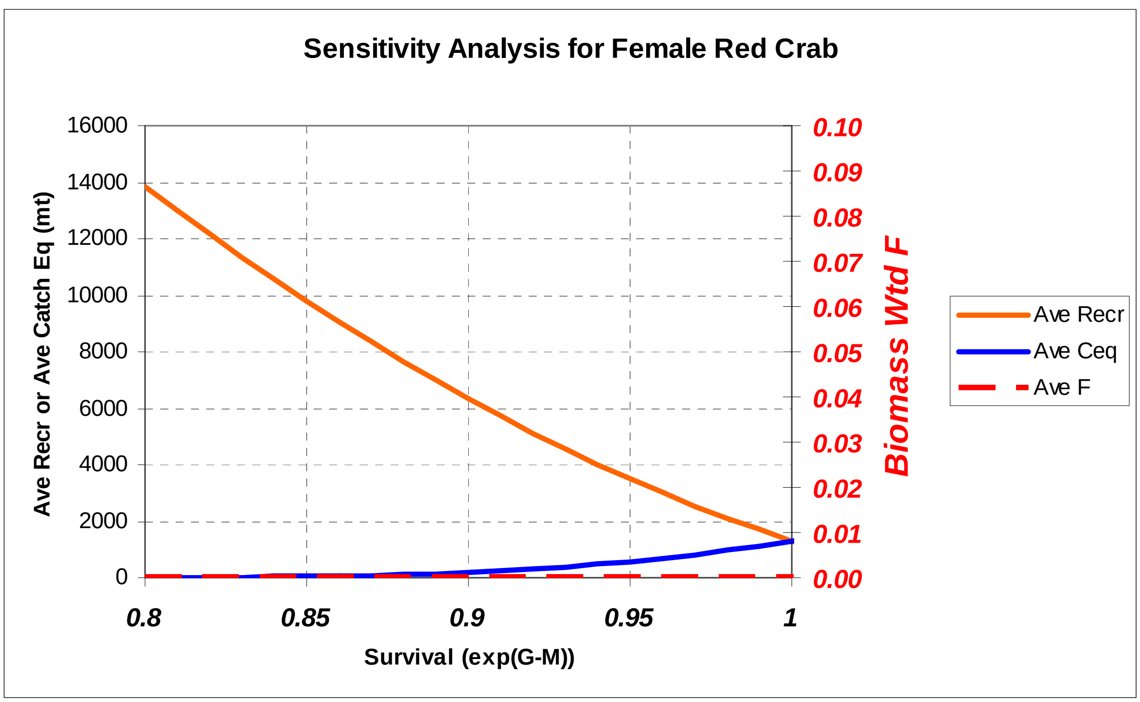

Fig. A3-2. Sensitivity analysis of recruitment, average F and equilibrium catch for female red crab to varying levels of survival rate.

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