Draft working paper for predissemination peer review only

Working Paper
February 23, 2010

Transition Discard Adaptive Alpha Method

J. Michael Lanning, PhD
Fisheries Statistics Office
Northeast Region
National Marine Fisheries Service

A working paper in support of the Discard Estimation Methodology Review

This information is distributed solely for the purpose of predissemination peer review. It has not been formally disseminated by NOAA. It does not represent any final agency determination or policy.

Background: Transition Discard Method

The trip discard rate for the observed trips is generated with a cumulative method, the sum of the observed discards divided by the sum of the kept-all. This cumulative discard rate is then weighed by an initial seed-rate weight through a hybrid exponential weighting.

(1) $$\mathrm{TR_I} = \alpha^I \cdot SR + (1-\alpha^I) \cdot \frac{\sum_{i=0}^{I} d_i}{\sum_{i=0}^{I} kall_i}, \text{ I from 1 to } \mathrm{I_{max}}$$

where

TR	=	Transition rate
I	=	Number of observed trips
α	=	Exponential weight or smoothing parameter
SR	=	Seed rate
d	=	Observed discard
kall	=	Observed kept-all

Mathematically this can be rewritten as:

(2) $$\mathrm{TR_I} = \mathrm{DR_I} + \alpha^I \left(SR - \mathrm{DR_I}\right)$$

where

(3) $$\mathrm{DR_I} = \frac{\sum_{i=0}^{I} d_i}{\sum_{i=0}^{I} kall_i}$$

is the cumulative discard rate at observation I.

The transition rate, TR, is seen to be equal to the cumulative discard rate, DR, plus a fraction (α^I) of the difference between the seed rate and the cumulative discard. As the number of observations, I, increases, the fraction (α^I) will decrease towards zero, and the transition rate will shift towards the cumulative discard rate. Of particular interest, is the choice of the exponential weight or smoothing parameter α.

There are several methods for automatically choosing the alpha value in simple exponential smoothing forecast models. Based on the errors of previous time periods, these are called Adaptive Response-Rate Exponential Smoothing (ARRES) methods. As an example, a method proposed by Trigg and Leach will be illustrated as it might be applied to the transition discard rate method.

Adaptive Response-Rate Exponential Smoothing

The ARRES method proposed by Trigg and Leach increases the alpha when errors are high (rapidly changing or systematic under/over forecasting) and decreases the alpha when errors are low (slowly changing).

The equation for single exponential smoothing is

(4) $$F_i = \alpha A_{i-1} + (1-\alpha)F_{i-1}$$

or

(5) $$F_i = F_{i-1} + \alpha(A_{i-1} - F_{i-1})$$

where

F_i	=	Exponentially smoothed forecast for observation i
F_{i-1}	=	Exponentially smoothed forecast for prior observation
A_i	=	Actual in the prior period
α	=	Exponential weight or smoothing parameter

The error at observation I is then

(6) $$Error_i = (A_i - F_i)$$

The adaptive response rate or alpha at observation I, α_i, is calculated as a weighted ratio of the absolute ratio of the smoothed forecast error to the smoothed absolute error as shown in equations 7 through 9.

(7) $$a_i = \left|\frac{SAD_i}{MAD_i}\right|$$

(8) $$SAD_i = \beta * (Error_i) + (1-\beta)SAD_{i-1}$$

(9) $$MAD_i = \beta * |Error_i| + (1-\beta)MAD_{i-1}$$

where

α_i	=	Alpha in forecasting period i+1
β	=	Beta, a smoothing parameter typically chosen to be 0.2
MAD_i	=	Exponentially weighted mean absolute forecast error
SAD_i	=	Exponentially weighted mean forecast error

The alpha of equation 5 is then replaced by the value of equation 7, and the single exponential smoothing becomes

(10) $$F_i = F_{\text{i-1}} + \alpha_{i-1}(A_{\text{i-1}} - F_{\text{i-1}})$$

or

(11) $$F_i = F_{\text{i-1}} + \alpha_{i-1} * Error_{\text{i-1}}$$

In applying this method to the transition rate, the actual rate, A, will equal the cumulative discard ratio, DR. The seed rate, SR, and an initial alpha value equal to one can be used to initialize the ARRES method.

(12) $$A_1 = SR$$

(13) $$\alpha_1 = 1$$

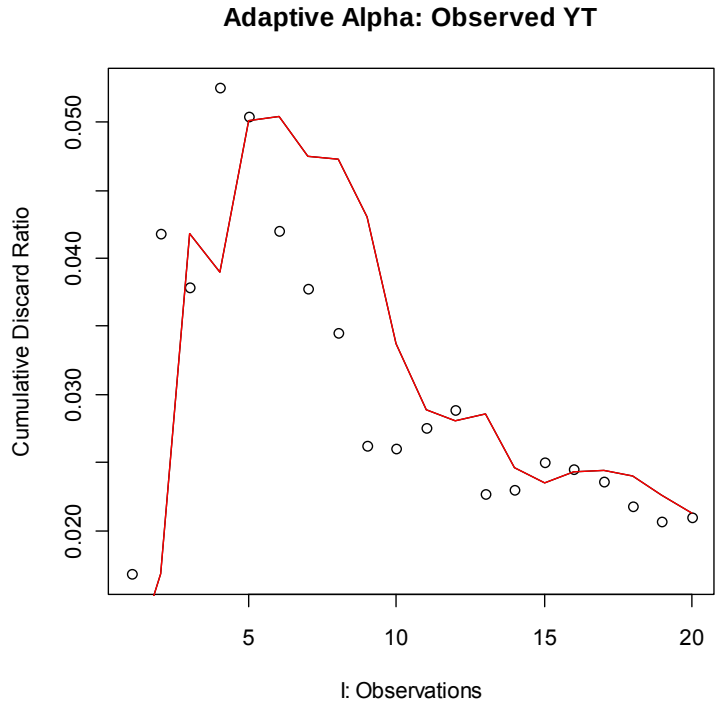
Example

As an example, non-regulatory large mesh yellowtail flounder, YT, discards in the Georges Bank YT stock area during the fishing year 2008 groundfish observer data from Observer Database Management System are used to illustrate adaptive alpha method. The forecasts and changing smoothing parameter can be viewed in Table 1 with the forecast line, F, plotted with the individual cumulative discard ratio, DR, in Figure 1.

Table 1: Example of Adaptive Alpha Rates and Forecasts SR = 0.1

Observation	Cumulative Discard Ratio	Beta =	0.2	Alpha	Forecast	Error
I	DR	SAD	MAD	α	F	
1	0.0168	0.0014	0.0014	1.000	0.01	0.006777
2	0.0419	0.0061	0.0061	1.000	0.016777	0.025084
3	0.0379	0.0041	0.0057	0.719	0.041861	-0.004
4	0.0525	0.0060	0.0073	0.824	0.038989	0.013534
5	0.0504	0.0048	0.0059	0.825	0.050137	0.000309
6	0.0421	0.0022	0.0064	0.348	0.050392	-0.00831
7	0.0378	-0.0002	0.0070	0.024	0.047501	-0.00969
8	0.0345	-0.0027	0.0082	0.329	0.047264	-0.01272
9	0.0263	-0.0055	0.0099	0.557	0.043084	-0.01681
10	0.0261	-0.0059	0.0094	0.629	0.033723	-0.00765
11	0.0275	-0.0050	0.0078	0.642	0.028917	-0.00137
12	0.0289	-0.0038	0.0064	0.598	0.028038	0.000853
13	0.0227	-0.0043	0.0063	0.673	0.028549	-0.00587
14	0.0230	-0.0037	0.0054	0.692	0.024597	-0.00159
15	0.0250	-0.0027	0.0046	0.582	0.023494	0.001492
16	0.0245	-0.0021	0.0037	0.571	0.024363	0.000133
17	0.0237	-0.0018	0.0031	0.593	0.024439	-0.00078
18	0.0218	-0.0019	0.0029	0.653	0.023975	-0.00216
19	0.0207	-0.0019	0.0027	0.701	0.022567	-0.0019
20	0.0210	-0.0016	0.0022	0.708	0.021236	-0.00025

Figure 1:



Comments

As a ratio of averages,

$$\textbf{(14)} \qquad \mathrm{DR_I} = \frac{\sum_{i=0}^{I} d_i}{\sum_{i=0}^{I} kall_i} = \frac{\frac{\sum_{i=0}^{I} d_i}{I}}{\frac{\sum_{i=0}^{I} kall_i}{I}}$$

the cumulative discard rate is expected to reach a stable value over time with increasing observations, I. The number of observations for this to occur is dependent on the parent population distributions of both the discards and the kept-all including the correlation between both groups. Based on the Central Limit Theorem and the possible skewness of the distributions, the DR can reach a stable value anywhere from 20 to 60 observations. This is a rule of thumb, and actual performance may vary.

Typically the sample size is chosen based on an acceptable error around the given estimation. In the Transition Discard Method, an alpha can then calculated based on the expected sample size needed.

Currently, many representative estimates for certain stratum groups needed for the discard ratio are small or missing. As such, the error estimates will only be available as the data is collected.

The ARRES method treats the Transition Discard Method as a forecast model allowing the alpha smoothing parameter to be self regulating with no need for an initial value to be chosen. The method is also a one step iteration that is easily calculated and/or programmed.

One weakness is that the adaptive alpha is based on the relative ratio between the exponentially smooth errors given in equations 8 and 9. As the DR trends to a point estimate over observations, alpha smoothing parameter will not trend towards zero allowing the user to drop the transition method for just the in season DR. An alternate criteria for dropping the model will be needed.

References

Trigg and Leach: "Exponential Smoothing with an Adaptive Response Rate." Operational Research Quarterly 18 (1967), pp 53-59